

Package ‘EFA.dimensions’

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Description Functions for eleven procedures for determining the number of factors, including functions for parallel analysis and the minimum average partial test. There are also functions for conducting principal components analysis, principal axis factor analysis, maximum likelihood factor analysis, image factor analysis, and extension factor analysis, all of which can take raw data or correlation matrices as input and with options for conducting the analyses using Pearson correlations, Kendall correlations, Spearman correlations, gamma correlations, or polychoric correlations. Varimax rotation, promax rotation, and Procrustes rotations can be performed. Additional functions focus on the factorability of a correlation matrix, the congruences between factors from different datasets, the assessment of local independence, the assessment of factor solution complexity, internal consistency, and for correcting Pearson correlation coefficients for attenuation due to unreliability.

Auerswald & Moshagen (2019, ISSN:1939-1463);
Field, Miles, & Field (2012, ISBN:978-1-4462-0045-2);
Mulaik (2010, ISBN:978-1-4200-9981-2);
O'Connor (2000, <[doi:10.3758/bf03200807](https://doi.org/10.3758/bf03200807)>);
O'Connor (2001, ISSN:0146-6216).

Imports stats, psych, polycor, EFAtools, utils, mirt, GPArotation

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EFA.dimensions-package

EFA.dimensions

Description

This package provides exploratory factor analysis-related functions for assessing dimensionality.

There are 11 functions for determining the number of factors (DIMTESTS, EMPKC, HULL, MAP, NEVALSGT1, PARALLEL, RAWPAR, ROOTFIT, SALIENT, SCREE_PLOT, SESCREE, and SMT).

There is a principal components analysis function (PCA), and an exploratory factor analysis function (EFA) with 9 possible factor extraction methods.

There are 15 possible factor rotation methods that can be used with PCA and EFA.

The analyses can be conducted using raw data or correlation matrices as input.

The analyses can be conducted using Pearson correlations, Kendall correlations, Spearman correlations, Goodman-Kruskal gamma correlations (Thompson, 2006), or polychoric correlations (using the psych and polychor packages).

Additional functions focus on the factorability of a correlation matrix (FACTORABILITY), the congruences between factors from different datasets (CONGRUENCE), the assessment of local independence (LOCALDEP), the assessment of factor solution complexity (COMPLEXITY), internal consistency (INTERNAL_CONSISTENCY), and for correcting Pearson correlation coefficients for attenuation due to unreliability (CORRECTED_CORRELS).

References

Auerswald, M., & Moshagen, M. (2019). How to determine the number of factors to retain in exploratory factor analysis: A comparison of extraction methods under realistic conditions. *Psychological Methods*, 24(4), 468-491.

Field, A., Miles, J., & Field, Z. (2012). *Discovering statistics using R*. Los Angeles, CA: Sage. ISBN:978-1-4462-0045-2

Mulaik, S. A. (2010). *Foundations of factor analysis* (2nd ed.). Boca Raton, FL: Chapman and Hall/CRC Press, Taylor & Francis Group.

O'Connor, B. P. (2000). SPSS and SAS programs for determining the number of components using parallel analysis and Velicer's MAP test. *Behavior Research Methods, Instrumentation, and Computers*, 32, 396-402.

O'Connor, B. P. (2000). SPSS and SAS programs for determining the number of components using parallel analysis and Velicer's MAP test. *Behavior Research Methods, Instrumentation, and Computers*, 32, 396-402.

Sellbom, M., & Tellegen, A. (2019). Factor analysis in psychological assessment research: Common pitfalls and recommendations. *Psychological Assessment*, 31(12), 1428-1441. <https://doi.org/10.1037/pas0000623>

Watts, A. L., Greene, A. L., Ringwald, W., Forbes, M. K., Brandes, C. M., Levin-Aspenson, H. F., & Delawalla, C. (2023). Factor analysis in personality disorders research: Modern issues and illustrations of practical recommendations. *Personality Disorders: Theory, Research, and Treatment*, 14(1), 105-117. <https://doi.org/10.1037/per0000581>

Description

Provides Hoffman's (1978) complexity coefficient for each item and (optionally) the percent complexity in the factor solution using the procedure and code provided by Pettersson and Turkheimer (2014).

Usage

```
COMPLEXITY(loadings, percent=TRUE, degree.change=100, averaging.value=100, verbose=TRUE)
```

Arguments

loadings	The factor loading matrix.
percent	(logical) Should the percent complexity be computed? The default = TRUE.
degree.change	If percent=TRUE, the number of incremental changes toward simple structure. The default = 100.
averaging.value	If percent=TRUE, the number of repeats per unit of degree change. The default = 100.
verbose	(logical) Should detailed results be displayed in console? The default = TRUE.

Details

This function provides Hoffman's (1978) complexity coefficient for each item and (optionally) the percent complexity in the factor solution using the procedure and code provided by Pettersson and Turkheimer (2014). For the percent complexity coefficient, values closer to zero indicate greater consistency with simple structure.

Value

A list with the following elements:

comp_rows	The complexity coefficient for each item
percent	The percent complexity in the factor solution

Author(s)

Brian P. O'Connor

References

Hofmann, R. J. (1978). Complexity and simplicity as objective indices descriptive of factor solutions. *Multivariate Behavioral Research*, 13, 247-250.

Pettersson E, Turkheimer E. (2010) Item selection, evaluation, and simple structure in personality data. *Journal of research in personality*, 44(4), 407-420.

Pettersson, E., & Turkheimer, E. (2014). Self-reported personality pathology has complex structure and imposing simple structure degrades test information. *Multivariate Behavioral Research*, 49(4), 372-389.

Examples

```
# the Harman (1967) correlation matrix
PCAoutput <- PCA(data_Harman, Nfactors = 2, Ncases = 305, rotation='promax', verbose=FALSE)
COMPLEXITY(loadings=PCAoutput$structure, verbose=TRUE)

# Rosenberg Self-Esteem scale items
PCAoutput <- PCA(data_RSE, Nfactors = 2, rotation='promax', verbose=FALSE)
COMPLEXITY(loadings=PCAoutput$structure, verbose=TRUE)

# NEO-PI-R scales
PCAoutput <- PCA(data_NEOPIR, Nfactors = 5, rotation='promax', verbose=FALSE)
COMPLEXITY(loadings=PCAoutput$structure, verbose=TRUE)
```

CONGRUENCE

Factor solution congruence

Description

Aligns two factor loading matrices and computes the factor solution congruence and the root mean square residual.

Usage

```
CONGRUENCE(target, loadings, verbose)
```

Arguments

target	The target loading matrix.
loadings	The loading matrix that will be aligned with the target.
verbose	Should detailed results be displayed in console? TRUE (default) or FALSE

Details

The function first searches for the alignment of the factors from the two loading matrices that has the highest factor solution congruence. It then aligns the factors in "loadings" with the factors in "target" without changing the loadings. The alignment is based solely on the positions and directions of the factors. The function then produces the Tucker-Wrigley-Neuhaus factor solution congruence coefficient as an index of the degree of similarity between between the aligned loading matrices (see Guadagnoli & Velicer, 1991; and ten Berge, 1986, for reviews).

An investigation by Lorenzo-Seva and ten Berge (2006) resulted in the following conclusion: A congruence coefficient "value in the range .85.94 corresponds to a fair similarity, while a value higher than .95 implies that the two factors or components compared can be considered equal."

Value

A list with the following elements:

rcBefore	The factor solution congruence before factor alignment
rcAfter	The factor solution congruence after factor alignment
rcFactors	The congruence for each factor
rmsr	The root mean square residual
residmat	The residual matrix
loadingsNew	The aligned loading matrix

Author(s)

Brian P. O'Connor

References

Guadagnoli, E., & Velicer, W. (1991). A comparison of pattern matching indices. *Multivariate Behavior Research*, 26, 323-343.

ten Berge, J. M. F. (1986). Some relationships between descriptive comparisons of components from different studies. *Multivariate Behavioral Research*, 21, 29-40.

Lorenzo-Seva, U., & ten Berge, J. M. F. (2006). Tucker's congruence coefficient as a meaningful index of factor similarity. *Methodology: European Journal of Research Methods for the Behavioral and Social Sciences*, 2(2), 5764.

Examples

```
# Rosenberg Self-Esteem scale items
loadings <- PCA(data_RSE[1:150,], corkind='pearson', Nfactors = 3,
                  rotation='varimax', verbose=FALSE)

target <- PCA(data_RSE[151:300,], corkind='pearson', Nfactors = 3,
                  rotation='varimax', verbose=FALSE)
CONGRUENCE(target = target$loadingsROT, loadings = loadings$loadingsROT, verbose=TRUE)

# NEO-PI-R scales
loadings <- PCA(data_NEOPIR[1:500,], corkind='pearson', Nfactors = 3,
                  rotation='varimax', verbose=FALSE)

target <- PCA(data_NEOPIR[501:1000,], corkind='pearson', Nfactors = 3,
                  rotation='varimax', verbose=FALSE)
CONGRUENCE(target$loadingsROT, loadings$loadingsROT, verbose=TRUE)
```

CORRECTED_CORRELS	<i>Corrected Pearson correlation coefficients</i>
-------------------	---

Description

Corrects Pearson correlation coefficients for attenuation due to unreliability and tests for overall differences between the original and corrected correlation matrices.

Usage

```
CORRECTED_CORRELS(cormat, Ncases = 100, alphas,
                     systematic=FALSE, Nperms = 1000, verbose=TRUE)
```

Arguments

cormat	A square correlation matrix.
Ncases	The number of cases upon which cormat is based. Example: Ncases = 250
alphas	A vector of reliability coefficients (e.g., Cronbach alpha values) for the variables in cormat, in the same order as the variables in cormat. Example: alphas <- c(.81, .95, .64, .49, .36)
systematic	The kind of permutations for the tests for significance differences between the uncorrected and corrected correlation matrices. The options are TRUE (for all possible permutations), or FALSE (the default), in which case random matrix permutations will be conducted.
Nperms	(optional) The number of random matrix permutation for when systematic = FALSE. Example: Nperms = 1000
verbose	(optional) Should detailed results be displayed in console? TRUE (default) or FALSE

Details

This function uses the simple, traditional formula for correcting correlations for attenuation due to unreliability, as described in Schmidt and Hunter (1996), Schmitt (1996), and Trafimow (2016).

Spearman's rho and Kendall's tau correlations between the values in the entered and corrected correlation matrices are provided when there are three or more variables. These are rank correlations that do not assume that similarity increases are linear and which are more robust to outliers than are Pearson correlations. The corresponding permutation significance tests of overall equality between the entered and corrected correlation matrices are "Mantel" tests.

The Steiger (1980) and Jennrich (1970) tests of the overall equality of the entered and corrected correlation matrices are conducted using functions from the psych package (Revelle, 2023).

See the MatrixCorrelation package for additional tests of differences between correlation matrices.

Value

A list with the following elements:

cormat	The entered correlation matrix
alphas	The entered alphas
corxd	The matrix of corrected correlations
resids	The differences between the entered and corrected correlations
data_in_rows	The input data and results in row format
cor_spearman	The Spearman's rho correlation between the entered and corrected correlations
cor_kendall	The Kendall's tau correlation between the entered and corrected correlations
Steiger_test	Results of the Steiger test for equality of the entered and corrected correlation matrices
Jennrich_test	Results of the Jennrich test for equality of the entered and corrected correlation matrices

Author(s)

Brian P. O'Connor

References

Jennrich, R. I. (1970) An asymptotic test for the equality of two correlation matrices. *Journal of the American Statistical Association*, 65, 904-912.

Revelle, W. (2023). psych: Procedures for Psychological, Psychometric, and Personality Research. R package version 2.3.6, <https://CRAN.R-project.org/package=psych>

Schmidt, F. L., & Hunter, J. E. (1996). Measurement error in psychological research: Lessons from 26 research scenarios. *Psychological Methods*, 1(2), 199223.

Schmitt, N. (1996). Uses and abuses of coefficient alpha. *Psychological Assessment*, 8(4), 350353.

Steiger, J. H. (1980). Testing pattern hypotheses on correlation matrices: Alternative statistics and some empirical results. *Multivariate Behavioral Research*, 15, 335-352.

Trafimow, D. (2016). The attenuation of correlation coefficients: A statistical literacy issue. *Teaching Statistics*, 38(1), 2528.

Examples

```
# data from Schmitt (1996)
cormat <- '
1.0
.70  1.0
.51  .47  1.0
.52  .55  .45  1.0
.32  .35  .31  .28  1.0'
```

```
cormat_noms <- c('v1', 'v2', 'v3', 'v4', 'v5')
cormat <- data.matrix( read.table(text=cormat, fill=TRUE, col.names=cormat_noms,
                                row.names=cormat_noms ))
cormat[upper.tri(cormat)] <- t(cormat)[upper.tri(cormat)]

alphas <- c(.81, .95, .64, .49, .36)

CORRECTED_CORRELS(cormat=cormat, alphas=alphas,
                    Nperms = 10000, systematic=FALSE, verbose=TRUE)
```

*data_Field**data_Field*

Description

A data frame with scores on 23 variables for 2571 cases. This is a simulated dataset that has the exact same correlational structure as the "R Anxiety Questionnaire" data used by Field et al. (2012) in their chapter on Exploratory Factor Analysis.

Usage

```
data(data_Field)
```

Source

Field, A., Miles, J., & Field, Z. (2012). *Discovering statistics using R*. Los Angeles, CA: Sage.

Examples

```
# MAP test
MAP(data_Field, corkind='pearson', verbose=TRUE)

# DIMTESTS
DIMTESTS(data_Field, corkind='pearson',
          tests = c('CD','EMPKC','HULL','RAWPAR','NEVALSGT1'), display=2)

# principal components analysis
PCA(data_Field, corkind='pearson', Nfactors=4, rotation='none', verbose=TRUE)
```

data_Harman

Correlation matrix from Harman (1967, p. 80).

Description

The correlation matrix for eight physical variables for 305 cases from Harman (1967, p. 80).

Usage

```
data(data_Harman)
```

References

Harman, H. H. (1967). *Modern factor analysis* (2nd. ed.). Chicago: University of Chicago Press.

Examples

```
# MAP test on the Harman correlation matrix
MAP(data_Harman, verbose=TRUE)

# DIMTESTS on the Harman correlation matrix
DIMTESTS(data_Harman, tests = c('EMPKC', 'HULL', 'RAWPAR', 'NEVALSGT1'), Ncases=305, display=2)

# parallel analysis of the Harman correlation matrix
RAWPAR(data_Harman, extraction='PCA', Ndatasets=100, percentile=95,
        Ncases=305, verbose=TRUE)
```

data_NEOPIR

data_NEOPIR

Description

A data frame with scores for 1000 cases on 30 variables that have the same intercorrelations as those for the Big 5 facets on pp. 100-101 of the NEO-PI-R manual (Costa & McCrae, 1992).

Usage

```
data(data_NEOPIR)
```

References

Costa, P. T., & McCrae, R. R. (1992). *Revised NEO personality inventory (NEO-PIR) and NEO five-factor inventory (NEO-FFI): Professional manual*. Odessa, FL: Psychological Assessment Resources.

Examples

```
# MAP test on the data_NEOPIR data
MAP(data_NEOPIR, corkind='pearson', verbose=TRUE)

# DIMTESTS on the data_NEOPIR data
DIMTESTS(data_NEOPIR, tests = c('EMPKC', 'HULL', 'RAWPAR', 'NEVALSGT1'), Ncases=1000, display=2)

# parallel analysis of the data_NEOPIR data
RAWPAR(data_NEOPIR, extraction='PCA', Ndatasets=100, percentile=95,
       corkind='pearson', verbose=TRUE)
```

data_RSE

Item-level dataset for the Rosenberg Self-Esteem scale

Description

A data frame with 300 observations on the 10 items from the Rosenberg Self-Esteem scale.

Usage

```
data(data_RSE)
```

References

Rosenberg, M. (1965). *Society and the adolescent self-image*. Princeton University Press.

Examples

```
# MAP test on the Rosenberg Self-Esteem Scale (RSE) item data
MAP(data_RSE, corkind='polychoric', verbose=TRUE)

# DIMTESTS on the Rosenberg Self-Esteem Scale (RSE) item data
DIMTESTS(data_RSE, tests = c('CD', 'EMPKC', 'HULL', 'RAWPAR', 'NEVALSGT1'), Ncases=1000, display=2)

# parallel analysis of the Rosenberg Self-Esteem Scale (RSE) item data
RAWPAR(data_RSE, extraction='PCA', Ndatasets=100, percentile=95,
       corkind='pearson', verbose=TRUE)
```

data_TabFid	<i>data_TabFid</i>
-------------	--------------------

Description

A data frame with scores for 340 cases on 44 Bem Sex Role Inventory items, used by Tabachnick & Fidell (2013, p. 656) in their chapter on exploratory factor analysis.

Usage

```
data(data_TabFid)
```

References

Tabachnick, B. G., & Fidell, L. S. (2013). *Using multivariate statistics*. New York, NY: Pearson.

Examples

```
# MAP test on the data_TabFid data
MAP(data_TabFid, corkind='pearson', verbose=TRUE)

# parallel analysis of the data_TabFid data
RAWPAR(data_TabFid, extraction='PCA', Ndatasets=100, percentile=95,
        corkind='pearson', verbose=TRUE)

# DIMTESTS on the data_TabFid data
DIMTESTS(data_TabFid, tests = c('EMPKC', 'HULL', 'RAWPAR'), corkind='pearson', display=1)

# principal axis factor analysis of the data_TabFid data
EFA(data_TabFid, corkind='pearson', extraction='paf', Nfactors = 5, iterpaf = 50,
      rotation='promax', ppower = 4, verbose=TRUE)
```

DIMTESTS	<i>Tests for the number of factors</i>
----------	--

Description

Conducts multiple tests for the number of factors

Usage

```
DIMTESTS(data, tests, corkind, Ncases, HULL_method, HULL_gof, HULL_cor_method,
          CD_cor_method, display)
```

Arguments

data	An all-numeric dataframe where the rows are cases & the columns are the variables, or a correlation matrix with ones on the diagonal. The function internally determines whether the data are a correlation matrix.
tests	A vector of the names of the tests for the number of factors that should be conducted. The possibilities are CD, EMPKC, HULL, MAP, NEVALSGT1, RAWPAR, SALIENT, SESCREE, SMT. If tests is not specified, then tests = c('EMPKC', 'HULL', 'RAWPAR') is used as the default.
corkind	The kind of correlation matrix to be used if data is not a correlation matrix. The options are 'pearson', 'kendall', 'spearman', 'gamma', and 'polychoric'. Required only if the entered data is not a correlation matrix.
Ncases	The number of cases. Required only if data is a correlation matrix.
HULL_method	From EFAtools: The estimation method to use. One of "PAF" (default), "ULS", or "ML", for principal axis factoring, unweighted least squares, and maximum likelihood
HULL_gof	From EFAtools: The goodness of fit index to use. Either "CAF" (default), "CFI", or "RMSEA", or any combination of them. If method = "PAF" is used, only the CAF can be used as goodness of fit index. For details on the CAF, see Lorenzo-Seva, Timmerman, and Kiers (2011).
HULL_cor_method	From EFAtools: The kind of correlation matrix to be used for the Hull method analyses. The options are 'pearson', 'kendall', and 'spearman'
CD_cor_method	From EFAtools: The kind of correlation matrix to be used for the CD method analyses. The options are 'pearson', 'kendall', and 'spearman'
display	The results to be displayed in the console: 0 = nothing; 1 = only the # of factors for each test; 2 (default) = detailed output for each test

Details

This is a convenience function for tests for the number of factors.

The HULL method option uses the HULL function (and its defaults) in the EFAtools package.

From Auerswald & Moshagen (2019):

"The Hull method (Lorenzo-Seva et al., 2011) is an approach based on the Hull heuristic used in other areas of model selection (e.g., Ceulemans & Kiers, 2006). Similar to nongraphical variants of Cattell's scree plot, the Hull method attempts to find an elbow as justification for the number of common factors. However, instead of using the eigenvalues relative to the number of factors, the Hull method relies on goodness-of-fit indices relative to the model degrees of freedom of the proposed model."

The CD (comparison data) method option uses the CD function (and its defaults) in the EFAtools package. The CD method can only be conducted on raw data and not on correlation matrices.

From Auerswald & Moshagen (2019):

"Ruscio and Roche (2012) suggested an approach that finds the number of factors by determining the solution that reproduces the pattern of eigenvalues best (comparison data, CD). CD takes previous factors into account by generating comparison data of a known factorial structure in an

iterative procedure. Initially, CD compares whether the simulated comparison data with one underlying factor ($j = 1$) reproduce the pattern of empirical eigenvalues significantly worse compared with a two-factor solution ($j + 1$). If this is the case, CD increases j until further improvements are nonsignificant or a preset maximum of factors is reached."

"No single extraction criterion performed best for every factor model. In unidimensional and orthogonal models, traditional PA, EKC, and Hull consistently displayed high hit rates even in small samples. Models with correlated factors were more challenging, where CD and SMT outperformed other methods, especially for shorter scales. Whereas the presence of cross-loadings generally increased accuracy, non-normality had virtually no effect on most criteria. We suggest researchers use a combination of SMT and either Hull, the EKC, or traditional PA, because the number of factors was almost always correctly retrieved if those methods converged. When the results of this combination rule are inconclusive, traditional PA, CD, and the EKC performed comparatively well. However, disagreement also suggests that factors will be harder to detect, increasing sample size requirements to $N \geq 500$."

The recommended tests for the number of factors are: EMPKC, HULL, and RAWPAR. The MAP test is also recommended for principal components analyses. Other possible methods (e.g., NEVALSGT1, SALIENT, SESCREE) are less well-validated and are included for research purposes.

Value

A list with the following elements:

dimtests	A matrix with the DIMTESTS results
Nfactors	DIMTESTS
	The number of factors according to the first test method specified in the "tests" vector

Author(s)

Brian P. O'Connor

References

Auerswald, M., & Moshagen, M. (2019). How to determine the number of factors to retain in exploratory factor analysis: A comparison of extraction methods under realistic conditions. *Psychological Methods*, 24(4), 468-491.

Lorenzo-Seva, U., Timmerman, M. E., & Kiers, H. A. (2011). The Hull method for selecting the number of common factors. *Multivariate Behavioral Research*, 46(2), 340-364.

O'Connor, B. P. (2000). SPSS and SAS programs for determining the number of components using parallel analysis and Velicer's MAP test. *Behavior Research Methods, Instrumentation, and Computers*, 32, 396-402.

Ruscio, J., & Roche, B. (2012). Determining the number of factors to retain in an exploratory factor analysis using comparison data of known factorial structure. *Psychological Assessment*, 24, 282292. doi: 10.1037/a0025697

Zwick, W. R., & Velicer, W. F. (1986). Comparison of five rules for determining the number of components to retain. *Psychological Bulletin*, 99, 432-442.

Examples

```
# the Harman (1967) correlation matrix
DIMTESTS(data_Harman, tests = c('EMPKC', 'HULL', 'RAWPAR'), corkind='pearson',
          Ncases = 305, display=2)

# Rosenberg Self-Esteem scale items, all possible DIMTESTS
DIMTESTS(data_RSE,
          tests = c('CD', 'EMPKC', 'HULL', 'MAP', 'NEVALSGT1', 'RAWPAR', 'SALIENT', 'SESCREE', 'SMT'),
          corkind='pearson', display=2)

# Rosenberg Self-Esteem scale items, using polychoric correlations
DIMTESTS(data_RSE, corkind='polychoric', display=2)

# NEO-PI-R scales
DIMTESTS(data_NEOPIR, tests = c('EMPKC', 'HULL', 'RAWPAR', 'NEVALSGT1'), display=2)
```

EFA

Exploratory factor analysis

Description

Exploratory factor analysis with multiple options for factor extraction and rotation

Usage

```
EFA(data, extraction = 'paf', corkind='pearson', Nfactors=NULL, Ncases=NULL, iterpaf=100,
     rotation='promax', ppower = 3, verbose=TRUE)
```

Arguments

data	An all-numeric dataframe where the rows are cases & the columns are the variables, or a correlation matrix with ones on the diagonal. The function internally determines whether the data are a correlation matrix.
extraction	The factor extraction method for the analysis. The options are 'paf' (the default), 'ml', 'image', 'minres', 'uls', 'ols', 'wls', 'gls', 'alpha', and 'fullinfo'.
corkind	The kind of correlation matrix to be used if data is not a correlation matrix. The options are 'pearson', 'kendall', 'spearman', 'gamma', and 'polychoric'. Required only if the entered data is not a correlation matrix.
Nfactors	The number of factors to extract. If not specified, then the EMPKC procedure will be used to determine the number of factors.
Ncases	The number of cases. Required only if data is a correlation matrix.
iterpaf	The maximum number of iterations for paf.

rotation	The factor rotation method for the analysis. The orthogonal rotation options are: 'varimax' (the default), 'quartimax', 'bentlerT', 'equamax', 'geominT', 'bifactorT', 'entropy', and 'none'. The oblique rotation options are: 'promax' (the default), 'quartimin', 'oblimin', 'oblimax', 'simplimax', 'bentlerQ', 'geominQ', 'bifactorQ', and 'none'.
ppower	The power value to be used in a promax rotation (required only if rotation = 'promax'). Suggested value: 3
verbose	Should detailed results be displayed in console? TRUE (default) or FALSE

Details

The factor extraction computations for the following methods are conducted using the psych package (Revelle, 2023): 'minres', 'uls', 'ols', 'wls', 'gls', and 'alpha'.

The factor extraction computations for 'fullinfo' are conducted using the mirt package (Chalmers, 2012). Full-information methods are considered more appropriate for item-level data than other factor extraction methods (Wirth & Edwards, 2007).

The factor rotation computations for the following methods are conducted using the GPArotation package (Bernaards & Jennrich, 2005, 2023): 'quartimax', 'bentlerT', 'geominT', 'bifactorT', 'entropy', 'quartimin', 'oblimin', 'oblimax', 'simplimax', 'bentlerQ', 'geominQ', and 'bifactorQ'.

For factor extraction (see Mulaik, 2010, for a review):

- **paf** is for principal axis factor analysis
- **ml** is for maximum likelihood factor analysis
- **image** is for image factor analysis
- **minres** is for a minimum residual factor analysis (Revelle, 2023)
- **uls** is for an unweighted least squares factor analysis (Revelle, 2023)
- **ols** is for an ordinary least squares factor analysis (Revelle, 2023)
- **wls** is for a weighted least squares factor analysis (Revelle, 2023)
- **gls** is for a generalized weighted least squares factor analysis (Revelle, 2023)
- **alpha** is for an alpha factor analysis (Revelle, 2023)

For factor rotation (see Jennrich, 2018, for a review):

- **varimax** is an orthogonal rotation that maximizes the spread of loadings within factors, which facilitates the interpretation of factors
- **quartimax** is an orthogonal rotation that maximizes the spread of loadings for each variable across factors, which facilitates the interpretation of variables (Bernaards & Jennrich, 2023)
- **bentlerT** is an orthogonal rotation based on Bentler's invariant pattern simplicity criterion (Bernaards & Jennrich, 2023)
- **equamax** is an orthogonal rotation from the Crawford-Ferguson family (Bernaards & Jennrich, 2023)
- **geominT** is an orthogonal rotation (Bernaards & Jennrich, 2023)
- **bifactorT** is an orthogonal Jennrich and Bentler bifactor rotation (Bernaards & Jennrich, 2023)

- **entropy** is a minimum entropy orthogonal rotation (Bernaards & Jennrich, 2023)
- **promax** is an oblique rotation
- **quartimin** is an oblique rotation (Bernaards & Jennrich, 2023)
- **oblimin** is an oblique rotation (Bernaards & Jennrich, 2023)
- **oblimax** is an oblique rotation (Bernaards & Jennrich, 2023)
- **simplimax** is an oblique rotation (Bernaards & Jennrich, 2023)
- **bentlerQ** is an oblique rotation based on Bentler's invariant pattern simplicity criterion (Bernaards & Jennrich, 2023)
- **geominQ** is an oblique rotation (Bernaards & Jennrich, 2023)
- **bifactorQ** is an oblique Jennrich and Bentler biquartimin rotation (Bernaards & Jennrich, 2023)

Value

A list with the following elements:

loadingsNOROT	The unrotated factor loadings
loadingsROT	The rotated factor loadings
pattern	The pattern matrix
structure	The structure matrix
phi	The correlations between the factors
varexplNOROT1	The initial eigenvalues and total variance explained
varexplNOROT2	The eigenvalues and total variance explained after factor extraction (no rotation)
varexplROT	The rotation sums of squared loadings and total variance explained for the rotated loadings
cormat_reprod	The reproduced correlation matrix, based on the rotated loadings
fit_coefs	Model fit coefficients
chisqMODEL	The model chi squared
dfMODEL	The model degrees of freedom
pvalue	The model p-value
chisqNULL	The null model chi squared
dfNULL	The null model degrees of freedom
communalities	The unrotated factor solution communalities
uniquenesses	The unrotated factor solution uniquenesses

Author(s)

Brian P. O'Connor

References

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Watts, A. L., Greene, A. L., Ringwald, W., Forbes, M. K., Brandes, C. M., Levin-Aspenson, H. F., & Delawalla, C. (2023). Factor analysis in personality disorders research: Modern issues and illustrations of practical recommendations. *Personality Disorders: Theory, Research, and Treatment*, 14(1), 105-117. <https://doi.org/10.1037/pe0000581>

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Examples

```
# the Harman (1967) correlation matrix
EFA(data=data_Harman, extraction = 'paf', Nfactors=2, Ncases=305, rotation='oblimin', verbose=TRUE)

# Rosenberg Self-Esteem scale items, using ml extraction & bifactorQ rotation
EFA(data=data_RSE, extraction = 'ml', corkind='polychoric', Nfactors=2,
     rotation='bifactorQ', verbose=TRUE)

# Rosenberg Self-Esteem scale items, using full-information factor extraction
EFA(data=data_RSE, extraction = 'fullinfo', corkind='pearson', Nfactors=2,
     rotation='none', verbose=TRUE)

# NEO-PI-R scales
EFA(data=data_NEOPIR, extraction = 'minres', corkind='pearson', Nfactors=5,
     iterpaf=100, rotation='promax', ppower = 4, verbose=TRUE)
```

EFA.dimensions-defunct*Defunct functions*

Description

The functions below are defunct and have been removed and replaced.

Details

- PA_FA has been replaced by [EFA](#)
- MAXLIKE_FA has been replaced by [EFA](#)
- IMAGE_FA has been replaced by [EFA](#)

Author(s)

Brian P. O'Connor <brian.oconnor@ubc.ca>

EFA_SCORES*Exploratory factor analysis scores*

Description

Factor scores, and factor score indeterminacy coefficients, for exploratory factor analysis

Usage

```
EFA_SCORES(loadings=NULL, loadings_type='structure', data=NULL, cormat=NULL,  
corkind='pearson', phi=NULL, method = 'Thurstone', verbose = TRUE)
```

Arguments

loadings	The factor loadings. Required for all methods except PCA.
loadings_type	(optional) The kind of factor loadings. The options are 'structure' (the default) or 'pattern'. Use 'structure' for orthogonal loadings.
data	(optional) An all-numeric dataframe where the rows are cases & the columns are the variables. Required if factor scores for cases are desired.
cormat	(optional) The item/variable correlation matrix. Not required when "data" is provided.
corkind	(optional) The kind of correlation matrix to be used. The options are 'pearson', 'kendall', 'spearman', 'gamma', and 'polychoric'. The kind of correlation should be the same as the kind that was used to produce the "loadings".

phi	(optional) The factor correlations.
method	(optional) The method to be used for computing the factor scores (e.g., method = 'Thurstone'). The options are: <ul style="list-style-type: none"> • Thurstone for Thurstone's (1935) least squares regression approach (Grice Equation 5), which is conducted on the factor structure loadings • Harman for Harman's (1976) idealized variables (Grice Equation 10), which is conducted on the factor pattern loadings. • Bartlett for Bartlett's (1937) method (Grice Equation 9), which is conducted on the factor pattern loadings. • tenBerge for ten Berge et al's (1999) method, which is conducted on the factor pattern loadings and which generates scores with correlations that are identical to those in phi (Grice Equation 8) • Anderson for Anderson & Rubin (1956; see Gorsuch, 1983, p 265) scores, which is conducted on the factor pattern loadings and which is only appropriate for orthogonal factor models • PCA for unrotated principal component scores (requires only data or cor-mat).
verbose	(optional) Should detailed results be displayed in console? TRUE (default) or FALSE

Details

Before using factor scores, it is important to establish that there is an acceptable degree of "determinacy" for the computed factor scores (Grice, 2001; Waller, 2023).

The following descriptions of factor score indeterminacy are either taken directly from, or adapted from, Grice (2001):

"As early as the 1920s researchers recognized that, even if the correlations among a set of ability tests could be reduced to a subset of factors, the scores on these factors would be indeterminate (Wilson, 1928). In other words, an infinite number of ways for scoring the individuals on the factors could be derived that would be consistent with the same factor loadings. Under certain conditions, for instance, an individual with a high ranking on g (general intelligence), according to one set of factor scores, could receive a low ranking on the same common factor according to another set of factor scores, and the researcher would have no way of deciding which ranking is "true" based on the results of the factor analysis. As startling as this possibility seems, it is a fact of the mathematics of the common factor model.

The indeterminacy problem is not that the factor scores cannot be directly and appropriately computed; it is that an infinite number of sets of such scores can be created for the same analysis that will all be equally consistent with the factor loadings.

The degree of indeterminacy will not be equivalent across studies and is related to the ratio between the number of items and factors in a particular design (Meyer, 1973; Schonemann, 1971). It may also be related to the magnitude of the communalities (Gorsuch, 1983). Small amounts of indeterminacy are obviously desirable, and the con- sequences associated with a high degree of indeterminacy are extremely unsettling. Least palatable is the fact that if the maximum possible proportion of indeterminacy in the scores for a particular factor meets or exceeds 50 struct two orthogonal or negatively correlated sets of factor scores that will be equally consistent with the same factor loadings (Guttman, 1955).

MULTR & RSQR MULTR is the multiple correlation between each factor and the original variables (Green, 1976; Mulaik, 1976). MULTR ranges from 0 to 1, with high values being desirable, and indicates the maximum possible degree of determinacy for factor scores. Some authors have suggested that MULTR values should be substantially higher than .707 which, when squared, would equal .50. RSQR is the square of MULTR and represents the maximum proportion of determinacy.

MINCOR

the minimum correlation that could be obtained between two sets of equally valid factor scores for each factor (Guttman, 1955; Mulaik, 1976; Schonemann, 1971). This index ranges from -1 to +1. High positive values are desirable. When MINCOR is zero, then two sets of competing factor scores can be constructed for the same common factor that are orthogonal or even negatively correlated. MINCOR values approaching zero are distressing, and negative values are disastrous. MINCOR values of zero or less occur when MULTR $\leq .707$ (at least 50 indeterminacy). MULTR values that do not appreciably exceed .71 are therefore particularly problematic. High values that approach 1.0 indicate that the factors may be slightly indeterminate, but the infinite sets of factor scores that could be computed will yield highly similar rankings of the individuals. In other words, the practical impact of the indeterminacy is minimal. MINCOR is the "Guttman's Indeterminacy Index" that is provided by the `fsIndeterminacy` function in the `fungible` package.

VALIDITY

While the MULTR values represent the maximum correlation between the factor score estimates and the factors, the VALIDITY coefficients represent the actual correlations between the factor score estimates and their respective factors, which may be lower than MULTR. The VALIDITY coefficients may range from -1 to +1. They should be interpreted in the same manner as MULTR. Gorsuch (1983, p. 260) recommended values of at least .80, but much larger values ($> .90$) may be necessary if the factor score estimates are to serve as adequate substitutes for the factors themselves.

Correlational Accuracy

If the factor score estimates are adequate representations of the factors, then the correlations between the factor scores should be similar to the correlations between the factors."

Value

A list with the following elements:

FactorScores	The factor scores
FSCoef	The factor score coefficients (W)
MULTR	The multiple correlation between each factor and the original variables
RSQR	The square of MULTR, representing the maximum proportion of determinacy
MINCOR	Guttman's indeterminacy index, the minimum correlation that could be obtained between two sets of equally valid factor scores for each factor.
VALIDITY	The correlations between the factor score estimates and their respective factors
UNIVOCALITY	The extent to which the estimated factor scores are excessively or insufficiently correlated with other factors in the same analysis
FactorScore_Correls	The correlations between the factor scores
phi	The correlations between the factors
pattern	The pattern matrix
pattern	The structure matrix

Author(s)

Brian P. O'Connor

References

Anderson, R. D., & Rubin, H. (1956). Statistical inference in factor analysis. *Proceedings of the Third Berkeley Symposium of Mathematical Statistics and Probability*, 5, 111-150.

Bartlett, M. S. (1937). The statistical conception of mental factors. *British Journal of Psychology*, 28, 97-104.

Grice, J. (2001). Computing and evaluating factor scores. *Psychological Methods*, 6(4), 430-450.

Harman, H. H. (1976). *Modern factor analysis*. University of Chicago press.

ten Berge, J. M. F., Krijnen, W. P., Wansbeek, T., and Shapiro, A. (1999). Some new results on correlation-preserving factor scores prediction methods. *Linear Algebra and its Applications*, 289(1-3), 311-318.

Thurstone, L. L. (1935). *The vectors of mind*. Chicago: University of Chicago Press.

Waller, N. G. (2023). Breaking our silence on factor score indeterminacy. *Journal of Educational and Behavioral Statistics*, 48(2), 244-261.

Examples

```
efa_out <- EFA(data=data_RSE, extraction = 'ml', Nfactors=2, rotation='promax')

EFA_SCORES(loadings=efa_out$structure, loadings_type='structure', data=data_RSE,
            phi=efa_out$phi, method = 'tenBerge')

# PCA scores
EFA_SCORES(data=data_NEOPIR, method = 'PCA')
```

Description

A test for the number of common factors using the Empirical Kaiser Criterion method (Braeken & van Assen, 2017).

Usage

```
EMPKC(data, corkind='pearson', Ncases=NULL, verbose=TRUE)
```

Arguments

data	An all-numeric dataframe where the rows are cases & the columns are the variables, or a correlation matrix with ones on the diagonal. The function internally determines whether the data are a correlation matrix.
corkind	The kind of correlation matrix to be used if data is not a correlation matrix. The options are 'pearson', 'kendall', 'spearman', 'gamma', and 'polychoric'. Required only if the entered data is not a correlation matrix.
Ncases	The number of cases. Required only if data is a correlation matrix.
verbose	Should detailed results be displayed in console? TRUE (default) or FALSE

Details

The code for this function was adapted from the code provided by Auerswald & Moshagen (2019).

From Braeken & van Assen (2017):

"We developed a new factor retention method, the Empirical Kaiser Criterion, which is directly linked to statistical theory on eigenvalues and to researchers' goals to obtain reliable scales. EKC is easily visualized, and easy to compute and apply (no specialized software or simulations are needed). EKC can be seen as a sample-variant of the original Kaiser criterion (which is only effective at the population level), yet with a built-in empirical correction factor that is a function of the variables-to-sample-size ratio and the prior observed eigenvalues in the series. The links with statistical theory and practically relevant scales allowed us to derive conditions under which EKC accurately retrieves the number of acceptable scales, that is, sufficiently reliable scales and strong enough items.

"Our simulations verified our derivations, and showed that (a) EKC performs about as well as parallel analysis for data arising from the null, 1-factor, or orthogonal factors model; and (b) clearly outperforms parallel analysis for the specific case of oblique factors, particularly whenever inter-factor correlation is moderate to high and the number of variables per factor is small, which is characteristic of many applications these days. Moreover, additional simulations suggest that our method for predicting conditions of accurate factor retention also work for the more computer-intensive methods ... The ease-of-use and effectiveness of EKC make this method a prime candidate for replacing parallel analysis, and the original Kaiser criterion that, although it empirically does not perform too well, is still the number one method taught in introductory multivariate statistics courses and the default in many commercial software packages. Furthermore, the link to statistical theory opens up possibilities for generic power curves and sample size planning for exploratory factor analysis studies.

"Generally, the EKC accurately retrieved the number of factors in conditions whenever it was predicted to work well, and its performance was worse when it was not predicted to work well. More precisely, hit rate or power exceeded .8 in accordance with predictions under the null model, 1-factor model, the orthogonal factor model, and the oblique factor model with more than three variables per scale. Only in the case of minimal scales, that is, with three items per scale, did EKC sometimes not accurately retrieve the number of factors as predicted; dropping the restriction that eigenvalues should exceed 1 then mended EKC's performance. A general guideline for application that can be derived from our results (and would not need a study-specific power study), is that EKC will accurately retrieve the number of factors in samples of at least 100 persons, when there is no factor, one practically relevant scale, or up to five practically relevant uncorrelated scales with a reliability of at least .8." (pp. 463-464)

From Auerswald & Moshagen (2019):

"The Empirical Kaiser Criterion (EKC; Braeken & van Assen, 2017) is an approach that incorporates random sample variations of the eigenvalues in Kaiser's criterion. On a population level, the criterion is equivalent to Kaiser's criterion and extractions all factors with associated eigenvalues of the correlation matrix greater than one. However, on a sample level, the criterion takes the distribution of eigenvalues for normally distributed data into account." (p. 474)

Value

The number of factors according to the EMPKC test.

Author(s)

Brian P. O'Connor

References

Auerswald, M., & Moshagen, M. (2019). How to determine the number of factors to retain in exploratory factor analysis: A comparison of extraction methods under realistic conditions. *Psychological Methods*, 24(4), 468-491.

Braeken, J., & van Assen, M. A. (2017). An empirical Kaiser criterion. *Psychological Methods*, 22, 450 - 466.

Examples

```
# the Harman (1967) correlation matrix
EMPKC(data_Harman, Ncases = 305)

# Rosenberg Self-Esteem scale items, using polychoric correlations
EMPKC(data_RSE, corkind='polychoric')

# NEO-PI-R scales
EMPKC(data_NEOPIR)
```

Description

Extension factor analysis, which provides correlations between nonfactored items and the factors that exist in a set of core items. The extension item correlations are then used to decide which factor, if any, a prospective item belongs to.

Usage

```
EXTENSION_FA(data, Ncore, Next, higherorder, roottest,
             corkind,
             extraction, rotation,
             Nfactors, NfactorsHO,
             Ndatasets, percentile,
             salvalue, numsals,
             iterpaf, ppower,
             verbose, factormodel, rotate)
```

Arguments

data	An all-numeric dataframe where the rows are cases & the columns are the variables.
Ncore	An integer indicating the number of core variables. The function will run the factor analysis on the data that appear in column #1 to column #Ncore of the data matrix.
Next	An integer indicating the number of extension variables, if any. The function will run extension factor analyses on the remaining columns in data, i.e., using column #Ncore+1 to the last column in data. Enter zero if there are no extension variables.
higherorder	Should a higher-order factor analysis be conducted? The options are TRUE or FALSE.
roottest	The method for determining the number of factors. The options are: 'Nsalient' for number of salient loadings (see salvalue & numsals below); 'parallel' for parallel analysis (see Ndatasets & percentile below); 'MAP' for Velicer's minimum average partial test; 'SEscree' for the standard error scree test; 'nevals>1' for the number of eigenvalues > 1; and 'user' for a user-specified number of factors (see Nfactors & NfactorsHO below).
corkind	The kind of correlation matrix to be used. The options are 'pearson', 'kendall', 'spearman', 'gamma', and 'polychoric'.
extraction	The factor extraction method. The options are: 'PAF' for principal axis / common factor analysis; 'PCA' for principal components analysis; 'ML' for maximum likelihood.
rotation	The factor rotation method. The options are: 'promax', 'varimax', and 'none'.
Nfactors	An integer indicating the user-determined number of factors (required only if roottest = 'user').
NfactorsHO	An integer indicating the user-determined number of higher order factors (required only if roottest = 'user' and higherorder = TRUE).
Ndatasets	An integer indicating the # of random data sets for parallel analyses (required only if roottest = 'parallel').
percentile	An integer indicating the percentile from the distribution of parallel analysis random eigenvalues to be used in determining the # of factors (required only if roottest = 'parallel'). Suggested value: 95

salvalue	The minimum value for a loading to be considered salient (required only if roottest = 'Nsalient'). Suggested value: .40
numsals	The number of salient loadings required for the existence of a factor i.e., the number of loadings > or = to salvalue (see above) for the function to identify a factor. Required only if roottest = 'Nsalient'. Gorsuch (1995a, p. 545) suggests: 3
iterpaf	The maximum # of iterations for a principal axis / common factor analysis (required only if extraction = 'PAF'). Suggested value: 100
ppower	The power value to be used in a promax rotation (required only if rotation = 'promax'). Suggested value: 3
verbose	Should detailed results be displayed in console? TRUE (default) or FALSE
factormodel	(Deprecated.) Use 'extraction' instead.
rotate	(Deprecated.) Use 'rotation' instead.

Details

Traditional scale development statistics can produce results that are baffling or misunderstood by many users, which can lead to inappropriate substantive interpretations and item selection decisions. High internal consistencies do not indicate unidimensionality; item-total correlations are inflated because each item is correlated with its own error as well as the common variance among items; and the default number-of-eigenvalues-greater-than-one rule, followed by principal components analysis and varimax rotation, produces inflated loadings and the possible appearance of numerous uncorrelated factors for items that measure the same construct (Gorsuch, 1997a, 1997b). Concerned investigators may then neglect the higher order general factor in their data as they use misleading statistical output to trim items and fashion unidimensional scales.

These problems can be circumvented in exploratory factor analysis by using more appropriate factor analytic procedures and by using extension analysis as the basis for adding items to scales. Extension analysis provides correlations between nonfactored items and the factors that exist in a set of core items. The extension item correlations are then used to decide which factor, if any, a prospective item belongs to. The decisions are unbiased because factors are defined without being influenced by the extension items. One can also examine correlations between extension items and any higher order factor(s) in the core items. The end result is a comprehensive, undisturbed, and informative picture of the correlational structure that exists in a set of core items and of the potential contribution and location of additional items to the structure.

Extension analysis is rarely used, at least partly because of limited software availability. Furthermore, when it is used, both traditional extension analysis and its variants (e.g., correlations between estimated factor scores and extension items) are prone to the same problems as the procedures mentioned above (Gorsuch, 1997a, 1997b). However, Gorsuch (1997b) described how diagonal component analysis can be used to bypass the problems and uncover the noninflated and unbiased extension variable correlations – all without computing factor scores.

Value

A list with the following elements:

fits1	eigenvalues & fit coefficients for the first set of core variables
rff	factor intercorrelations

corelding	core variable loadings on the factors
extcorrel	extension variable correlations with the factors
fits2	eigenvalues & fit coefficients for the higher order factor analysis
rfflding	factor intercorrelations from the first factor analysis and the loadings on the higher order factor(s)
ldingsef	variable loadings on the lower order factors and their correlations with the higher order factor(s)
extsef	extension variable correlations with the lower order factor(s) and their correlations with the higher order factor(s)

Author(s)

Brian P. O'Connor

References

Dwyer, P. S. (1937) The determination of the factor loadings of a given test from the known factor loadings of other tests. *Psychometrika*, 3, 173-178.

Gorsuch, R. L. (1997a). Exploratory factor analysis: Its role in item analysis. *Journal of Personality Assessment*, 68, 532-560.

Gorsuch, R. L. (1997b). New procedure for extension analysis in exploratory factor analysis. *Educational and Psychological Measurement*, 57, 725-740.

Horn, J. L. (1973) On extension analysis and its relation to correlations between variables and factor scores. *Multivariate Behavioral Research*, 8(4), 477-489.

O'Connor, B. P. (2001). EXTENSION: SAS, SPSS, and MATLAB programs for extension analysis. *Applied Psychological Measurement*, 25, p. 88.

Examples

```
EXTENSION_FA(data_RSE, Ncore=7, Next=3, higherorder=TRUE,
roottest='MAP',
corkind='pearson',
extraction='PCA', rotation='promax',
Nfactors=2, NfactorsH0=1,
Ndatasets=100, percentile=95,
salvalue=.40, numsals=3,
iterpaf=200,
ppower=4,
verbose=TRUE)

EXTENSION_FA(data_NEOPIR, Ncore=12, Next=6, higherorder=TRUE,
roottest='MAP',
corkind='pearson',
extraction='PCA', rotation='promax',
Nfactors=4, NfactorsH0=1,
```

```
Ndatasets=100, percentile=95,
salvalue=.40, numsals=3,
iterpaf=200,
ppower=4,
verbose=TRUE)
```

FACTORABILITY	<i>Factorability of a correlation matrix</i>
---------------	--

Description

Three methods for assessing the factorability of a correlation matrix

Usage

```
FACTORABILITY(data, corkind='pearson', Ncases=NULL, verbose=TRUE)
```

Arguments

data	An all-numeric dataframe where the rows are cases & the columns are the variables, or a correlation matrix with ones on the diagonal. The function internally determines whether the data are a correlation matrix.
corkind	The kind of correlation matrix to be used if data is not a correlation matrix. The options are 'pearson', 'kendall', 'spearman', 'gamma', and 'polychoric'. Required only if the entered data is not a correlation matrix.
Ncases	The number of cases for a correlation matrix. Required only if the entered data is a correlation matrix.
verbose	Should detailed results be displayed in console? TRUE (default) or FALSE

Details

This function provides results from three methods of assessing whether a dataset or correlation matrix is suitable for factor analysis:

- 1 – whether the determinant of the correlation matrix is > 0.00001 ;
- 2 – Bartlett's test of whether a correlation matrix is significantly different an identity matrix; and
- 3 – the Kaiser-Meyer-Olkin measure of sampling adequacy.

Value

A list with the following elements:

chisq	The chi-squared value for Bartlett's test
df	The degrees of freedom for Bartlett's test
pvalue	The significance level for Bartlett's test
Rimage	The image correlation matrix
KMO	The overall KMO value
KMOvars	The KMO values for the variables

Author(s)

Brian P. O'Connor

References

Bartlett, M. S. (1951). The effect of standardization on a chi square approximation in factor analysis, *Biometrika*, 38, 337-344.

Cerny, C. A., & Kaiser, H. F. (1977). A study of a measure of sampling adequacy for factor-analytic correlation matrices. *Multivariate Behavioral Research*, 12(1), 43-47.

Dziuban, C. D., & Shirkey, E. C. (1974). When is a correlation matrix appropriate for factor analysis? *Psychological Bulletin*, 81, 358-361.

Kaiser, H. F., & Rice, J. (1974). Little Jiffy, Mark IV. *Educational and Psychological Measurement*, 34, 111-117.

Examples

```
FACTORABILITY(data_RSE, corkind='pearson')

FACTORABILITY(data_Field, corkind='pearson')
```

INTERNAL_CONSISTENCY *Internal consistency reliability coefficients*

Description

Internal consistency reliability coefficients

Usage

```
INTERNAL_CONSISTENCY(data, extraction = 'ML', reverse_these = NULL,
                      auto_reverse = TRUE, verbose=TRUE, factormodel)
```

Arguments

data An all-numeric dataframe where the rows are cases & the columns are the variables.

extraction (optional) The factor extraction method to be used in the omega computations. The options are: 'ML' for maximum likelihood (the default); and 'PAF' for principal axis / common factor analysis.

reverse_these (optional) A vector of the names of items that should be reverse-coded

auto_reverse (optional) Should reverse-coding of items be conducted when warranted? TRUE (default) or FALSE

verbose	(optional) Should detailed results be displayed in console? TRUE (default) or FALSE
factormodel	(Deprecated.) Use 'extraction' instead.

Details

When 'auto_reverse = TRUE', the item loadings on the first principal component are computed and items with negative loadings are reverse-coded.

If error messages are produced, try using 'auto_reverse = FALSE'.

If item names are provided for the 'reverse_these' argument, then auto_reverse is not conducted.

The following helpful descriptions of Cronbach's alpha and of omega total are direct quotes from McNeish (2017, pp. 414-417):

Cronbach's Alpha

"One can interpret the value of Cronbach's alpha in one of many different ways:

1. Cronbach's alpha is the correlation of the scale of interest with another scale of the same length that intends to measure the same construct, with different items, taken from the same hypothetical pool of items (Kline, 1986).
2. The square root of Cronbach's alpha is an estimate of the correlation between observed scores and true scores (Nunnally & Bernstein, 1994).
3. Cronbach's alpha is the proportion of the variance of the scale that can be attributed to a common source (DeVellis, 1991).
4. Cronbach's alpha is the average of all possible split-half reliabilities from the set of items (Pedhazur & Schmelkin, 1991). (It is important to note the correlation between the two parts is not the split half reliability, but is used to find the split half reliability found by the Spearman-Brown prophecy formula.)

Under certain assumptions, Cronbach's alpha is a consistent estimate of the population internal consistency; however, these assumptions are quite rigid and are precisely why methodologists have argued against the use of Cronbach's alpha.

The assumptions of Cronbach's alpha are:

1. The scale adheres to tau equivalence, i.e., that each item on a scale contributes equally to the total scale score. Tau equivalence tends to be unlikely for most scales that are used in empirical research some items strongly relate to the construct while some are more weakly related.
2. Scale items are on a continuous scale and normally distributed. Cronbach's alpha is largely based on the observed covariances (or correlations) between items. In most software implementations of Cronbach's alpha (such as in SAS and SPSS), these item covariances are calculated using a Pearson covariance matrix. A well-known assumption of Pearson covariance matrices is that all variables are continuous in nature. Otherwise, the elements of the matrix can be substantially biased downward. However, it is particularly common for psychological scales to contain items that are discrete (e.g., Likert or binary response scales), which violates this assumption. If discrete items are treated as continuous, the covariance estimates will be attenuated, which ultimately results in underestimation of Cronbach's alpha because the relations between items will appear smaller than they actually are. To accommodate items that are not on a continuous scale, the covariances between items can instead be estimated with a polychoric covariance (or correlation) matrix rather than with a Pearson covariance matrix. Polychoric covariance matrices assume that there is an underlying normal distribution to discrete responses.

3. The errors of the items do not covary. Correlated errors occur when sources other than the construct being measured cause item responses to be related to one another.
4. The scale is unidimensional. Though Cronbach's alpha is sometimes thought to be a measure of unidimensionality because its colloquial definition is that it measures how well items stick together, unidimensionality is an assumption that needs to be verified prior to calculating Cronbach's alpha rather than being the focus of what Cronbach's alpha measures. Internal consistency is necessary for unidimensionality but that internal consistency is not sufficient for demonstrating unidimensionality. That is, items that measure different things can still have a high degree of interrelatedness, so a large Cronbach's alpha value does not necessarily guarantee that the scale measures a single construct. As a result, violations of unidimensionality do not necessarily bias estimates of Cronbach's alpha. In the presence of a multidimensional scale, Cronbach's alpha may still estimate the interrelatedness of the items accurately and the interrelatedness of multidimensional items can in fact be quite high."

Omega total

"Omega total is an internal consistency coefficient that assumes that the scale is unidimensional. Omega estimates the reliability for the composite of items on the scale (which is conceptually similar to Cronbach's alpha). Under the assumption that the construct variance is constrained to 1 and that there are no error covariances, omega total is calculated from factor analysis output (loadings and error/uniqueness values). Tau equivalence is no longer assumed and the potentially differential contribution of each item to the scale must be assessed. Omega total is a more general version of Cronbach's alpha and actually subsumes Cronbach's alpha as a special case. More simply, if tau equivalence is met, omega total will yield the same result as Cronbach's alpha but omega total has the flexibility to accommodate congeneric scales, unlike Cronbach's alpha."

"There are multiple variations of omega including omega hierarchical, omega total, and what we will refer to as 'Revelles omega total.' Omega hierarchical is useful for scales that may not be truly unidimensional and may contain additional minor dimensions. Omega hierarchical attempts to parse out the variability attributable to subfactors and calculates reliability for a general factor that applies to all items."

The INTERNAL_CONSISTENCY function only provides omega total, as described above. See the "psych" package for omega hierarchical and for Revelles omega total.

Root Mean Square Residual (rmsr)

rmsr is an index of the overall badness-of-fit. It is the square root of the mean of the squared residuals (the residuals being the simple differences between original correlations and the correlations implied by the N-factor model). rmsr is 0 when there is perfect model fit. A value less than .08 is generally considered a good fit. The rmsr coefficient is included in the internal consistency output as an index of the degree of fit of a one-factor model to the item data.

Standardized Cronbach's Alpha

Standardized alpha should be used when items have different scale ranges, e.g., some items are 1-to-7, and other items are 1-to-4, or 1-to-100. Regular alpha is based on covariances, whereas standardized alpha is based on correlations, wherein the items have identical standard deviations. Items in different metrics should be standardized before computing scale scores.

Value

A list with the following elements:

int.consist_scale	A vector with the scale omega, Cronbach's alpha, standardized Cronbach's alpha, the mean of the off-diagonal correlations, the median of the off-diagonal correlations, and the rmsr fit coefficient for a 1-factor model
int.consist_dropped	A matrix of the int.consist_scale values for when each item, in turn, is int.consist_dropped from the analyses
item_stats	The item means, standard deviations, and item-total correlations
resp_opt_freqs	The response option frequencies
resp_opt_props	The response option proportions
new_data	The data that was used for the analyses, including any item reverse-codings

Author(s)

Brian P. O'Connor

References

Flora, D. B. (2020). Your coefficient alpha is probably wrong, but which coefficient omega is right? A tutorial on using R to obtain better reliability estimates. *Advances in Methods and Practices in Psychological Science*, 3(4), 484501.

McNeish, D. (2018). Thanks coefficient alpha, we'll take it from here. *Psychological Methods*, 23(3), 412433.

Revelle, W., & Condon, D. M. (2019). Reliability from alpha to omega: A tutorial. *Psychological Assessment*, 31(12), 13951411.

Examples

```
# Rosenberg Self-Esteem scale items -- without reverse-coding
INTERNAL_CONSISTENCY(data_RSE, extraction = 'PAF',
                      reverse_these = NULL, auto_reverse = FALSE, verbose=TRUE)

# Rosenberg Self-Esteem scale items -- with auto_reverse-coding
INTERNAL_CONSISTENCY(data_RSE, extraction = 'PAF',
                      reverse_these = NULL, auto_reverse = TRUE, verbose=TRUE)

# Rosenberg Self-Esteem scale items -- another way of reverse-coding
INTERNAL_CONSISTENCY(data_RSE, extraction = 'PAF',
                      reverse_these = c('Q1','Q2','Q4','Q6','Q7'), verbose=TRUE)
```

LOCALDEP

*Local dependence***Description**

Provides the residual correlations after partialling latent trait scores out of an inter-item correlation matrix, along with local dependence statistics.

Usage

```
LOCALDEP(data, corkind, item_type, thetas, theta_type, verbose)
```

Arguments

<code>data</code>	An all-numeric dataframe where the rows are cases & the columns are the variables.
<code>corkind</code>	The kind of correlation matrix to be used for the analyses. The options are 'pearson', 'kendall', 'spearman', 'gamma', and 'polychoric'. Required only if the entered data is not a correlation matrix.
<code>item_type</code>	(optional) The type of items for the IRT analyses. If <code>item_type</code> is not specified, then it is assumed that the items follow a graded or 2PL model. The options for <code>item_type</code> are those that can be used in the <code>mirt</code> function from the <code>mirt</code> package, which include 'Rasch', '2PL', '3PL', '3PLu', '4PL', 'graded', 'grsm', 'grsmIRT', 'gpcm', 'gpcmIRT', 'rsm', 'nominal', 'ideal', <code>item_type</code> 'ggum', among other possibilities.
<code>thetas</code>	(optional) A vector of the latent trait scores that will be partialled out of the item correlations and used in computing other local dependence statistics. If <code>thetas</code> are not supplied, then they will be estimated internally using the <code>fscores</code> function from the <code>mirt</code> package.
<code>theta_type</code>	(optional) If <code>thetas</code> are not supplied, then they will be estimated internally using the <code>fscores</code> function from the <code>mirt</code> package with the following options: "EAP" (default), "MAP", "ML", "WLE", "EAPsum", "plausible", and "classify".
<code>verbose</code>	Should detailed results be displayed in console? TRUE (default) or FALSE

Details

Item response theory models are based on the assumption that the items display local independence. The latent trait is presumed to be responsible for the associations between the items. Once the latent trait is partialled out, the residual correlations between pairs of items should be negligible. Local dependence exists when there is additional systematic covariance among the items. It can occur when pairs of items have highly similar content or between sequentially presented items in a test. Local dependence distorts IRT parameter estimates, it can artificially increase scale information, and it distorts the latent trait, which becomes too heavily defined by the locally dependent items. Examining the residual (partial) correlations is a preliminary, exploratory method of determining whether local dependence exists. The function also displays the local dependence Q3 statistic values described by Yen (1984), the X2 statistic values described by Chen and Thissen (1997), the G2

statistic values described by Chen and Thissen (1997), and the jack-knife statistic values described by Edwards et al. (2018). The Q3, X2, G2, and jack-knife statistic values are obtained using the mirt function from the mirt package (Chalmers, 2012).

Value

A list with the following elements:

correlations	The correlation matrix
residcor	The residualized (partial) correlation matrix
eigenvalues	The eigenvalues
resid_Q3	A matrix with the Q3 statistic values described by Yen (1984)
resid_LD	A matrix with the X2 statistic values described by Chen and Thissen (1997)
resid_LDG2	A matrix with the G2 statistic values described by Chen and Thissen (1997)
resid_JSI	A matrix with the jack-knife statistic values described by Edwards et al. (2018)
localdep_stats	All of the above local dependence statistics in long format

Author(s)

Brian P. O'Connor

References

Chalmers, R., P. (2012). mirt: A multidimensional item response theory package for the R environment. *Journal of Statistical Software*, 48(6), 1-29.

Chen, W. H. & Thissen, D. (1997). Local dependence indices for item pairs using item response theory. *Journal of Educational and Behavioral Statistics*, 22, 265-289.

Edwards, M. C., Houts, C. R. & Cai, L. (2018). A diagnostic procedure to detect departures from local independence in item response theory models. *Psychological Methods*, 23, 138-149.

Yen, W. (1984). Effects of local item dependence on the fit and equating performance of the three parameter logistic model. *Applied Psychological Measurement*, 8, 125-145.

Examples

```
# Rosenberg Self-Esteem scale items
LOCALDEP(data_RSE)
```

MAP*Velicer's minimum average partial (MAP) test*

Description

Velicer's minimum average partial (MAP) test for determining the number of components, which focuses on the common variance in a correlation matrix.

Usage

```
MAP(data, corkind, Ncases, verbose)
```

Arguments

data	An all-numeric dataframe where the rows are cases & the columns are the variables, or a correlation matrix with ones on the diagonal. The function internally determines whether the data are a correlation matrix.
corkind	The kind of correlation matrix to be used if data is not a correlation matrix. The options are 'pearson', 'kendall', 'spearman', 'gamma', and 'polychoric'. Required only if the entered data is not a correlation matrix.
Ncases	The number of cases. Required only if data is a correlation matrix.
verbose	Should detailed results be displayed in console? TRUE (default) or FALSE

Details

This method for determining the number of components focuses on the common variance in a correlation matrix. It involves a complete principal components analysis followed by the examination of a series of matrices of partial correlations. Specifically, on the first step, the first principal component is partialled out of the correlations between the variables of interest, and the average squared coefficient in the off-diagonals of the resulting partial correlation matrix is computed. On the second step, the first two principal components are partialled out of the original correlation matrix and the average squared partial correlation is again computed. These computations are conducted for k (the number of variables) minus one steps. The average squared partial correlations from these steps are then lined up, and the number of components is determined by the step number in the analyses that resulted in the lowest average squared partial correlation. The average squared coefficient in the original correlation matrix is also computed, and if this coefficient happens to be lower than the lowest average squared partial correlation, then no components should be extracted from the correlation matrix. Statistically, components are retained as long as the variance in the correlation matrix represents systematic variance. Components are no longer retained when there is proportionately more unsystematic variance than systematic variance (see O'Connor, 2000, p. 397).

Value

A list with the following elements:

totvareexplNOROT	The eigenvalues and total variance explained
avgsqrs	Velicer's average squared correlations
NfactorsMAP	The number of components according to the original (1976) MAP test
NfactorsMAP4	The number of components according to the revised (2000) MAP test

Author(s)

Brian P. O'Connor

References

O'Connor, B. P. (2000). SPSS and SAS programs for determining the number of components using parallel analysis and Velicer's MAP test. *Behavior Research Methods, Instrumentation, and Computers*, 32, 396-402.

Velicer, W. F. (1976). Determining the number of components from the matrix of partial correlations. *Psychometrika*, 41, 321-327.

Velicer, W. F., Eaton, C. A., and Fava, J. L. (2000). Construct explication through factor or component analysis: A review and evaluation of alternative procedures for determining the number of factors or components. In R. D. Goffin & E. Helmes, eds., *Problems and solutions in human assessment* (p.p. 41-71). Boston: Kluwer.

Examples

```
# the Harman (1967) correlation matrix
MAP(data_Harman, cokind='pearson', Ncases = 305, verbose=TRUE)

# Rosenberg Self-Esteem scale items, using Pearson correlations
MAP(data_RSE, cokind='pearson', verbose=TRUE)

# Rosenberg Self-Esteem scale items, using polychoric correlations
MAP(data_RSE, cokind='polychoric', verbose=TRUE)

# NEO-PI-R scales
MAP(data_NEOPIR, verbose=TRUE)
```

MISSING_INFO	<i>Missing value statistics</i>
--------------	---------------------------------

Description

Frequencies and proportions of missing values

Usage

MISSING_INFO(data, verbose)

Arguments

data	An all-numeric dataframe where the rows are cases & the columns are the variables.
verbose	(optional) Should detailed results be displayed in console? TRUE (default) or FALSE

Details

Provides the number of cases with each of N missing values (NA values), along with the proportions, cumulative proportions, and the cumulative Ns.

Value

A matrix with the following columns:

N_cases	The number of cases
N_missing	The number of missing values
Proportion	The proportion of missing values
Cum_Proportion	The cumulative proportion of missing values
Cum_N	The cumulative number of cases

Author(s)

Brian P. O'Connor

Examples

```
MISSING_INFO(airquality)

# add NA values to the Rosenberg Self-Esteem scale items, for illustration
data_RSE_missing <- data_RSE
data_RSE_missing[matrix(rbinom(prod(dim(data_RSE_missing)), size=1, prob=.3)==1,
                           nrow=dim(data_RSE_missing)[1])] <- NA

MISSING_INFO(data_RSE_missing)
```

Description

Returns the count of the number of eigenvalues greater than 1 in a correlation matrix. This value is often referred to as the "Kaiser", "Kaiser-Guttman", or "Guttman-Kaiser" rule for determining the number of components or factors in a correlation matrix.

Usage

```
NEVALSGT1(data, corkind, Ncases, verbose=TRUE)
```

Arguments

data	An all-numeric dataframe where the rows are cases & the columns are the variables, or a correlation matrix with ones on the diagonal. The function internally determines whether the data are a correlation matrix.
corkind	The kind of correlation matrix to be used if data is not a correlation matrix. The options are 'pearson', 'kendall', 'spearman', 'gamma', and 'polychoric'. Required only if the entered data is not a correlation matrix.
Ncases	The number of cases. Required only if data is a correlation matrix.
verbose	Should detailed results be displayed in console? TRUE (default) or FALSE

Details

The rationale for this traditional procedure for determining the number of components or factors is that a component with an eigenvalue of 1 accounts for as much variance as a single variable. Extracting components with eigenvalues of 1 or less than 1 would defeat the usual purpose of component and factor analyses. Furthermore, the reliability of a component will always be nonnegative when its eigenvalue is greater than 1. This rule is the default retention criteria in SPSS and SAS.

There are a number of problems with this rule of thumb. Monte Carlo investigations have found that its accuracy rate is not acceptably high (Zwick & Velicer, 1986)). The rule was originally intended to be an upper bound for the number of components to be retained, but it is most often used as the criterion to determine the exact number of components or factors. Guttman's original proof applies only to the population correlation matrix and the sampling error that occurs in specific samples results in the rule often overestimating the number of components. The rule is also considered overly mechanical, e.g., a component with an eigenvalue of 1.01 achieves factor status whereas a component with an eigenvalue of .999 does not.

This function is included in this package for curiosity and research purposes.

Value

A list with the following elements:

NfactorsNEVALSGT1

The number of eigenvalues greater than 1.

totvarexplNOROT

The eigenvalues and total variance explained

Author(s)

Brian P. O'Connor

References

Fabrigar, L. R., Wegener, D. T., MacCallum, R. C., & Strahan, E. J. (1999). Evaluating the use of exploratory factor analysis in psychological research. *Psychological Methods*, 4, 272-299.

Guttman, L. (1954). Some necessary conditions for common factor analysis. *Psychometrika*, 19, 149-161.

Hayton, J. C., Allen, D. G., Scarpello, V. (2004). Factor retention decisions in exploratory factor analysis: A tutorial on parallel analysis. *Organizational Research Methods*, 7, 191-205.

Kaiser, H. F. (1960). The application of electronic computer to factor analysis. *Educational and Psychological Measurement*, 20, 141-151.

Zwick, W. R., & Velicer, W. F. (1986). Comparison of five rules for determining the number of components to retain. *Psychological Bulletin*, 99, 432-442.

Examples

```
# the Harman (1967) correlation matrix
NEVALSGT1(data_Harman, corkind='pearson', Ncases = 305, verbose=TRUE)

# Rosenberg Self-Esteem scale items, using Pearson correlations
NEVALSGT1(data_RSE, corkind='pearson', verbose=TRUE)

# Rosenberg Self-Esteem scale items, using polychoric correlations
NEVALSGT1(data_RSE, corkind='polychoric', verbose=TRUE)

# NEO-PI-R scales
NEVALSGT1(data_NEOPIR, corkind='pearson', verbose=TRUE)
```

PARALLEL

Parallel analysis of eigenvalues (random data only)

Description

Generates eigenvalues and corresponding percentile values for random data sets with specified numbers of variables and cases.

Usage

```
PARALLEL(Nvars=20, Ncases=300, Ndatasets=100, extraction='PCA', percentile='95',
corkind='pearson', verbose=TRUE, factormodel)
```

Arguments

Nvars	The number of variables.
Ncases	The number of cases.
Ndatasets	An integer indicating the # of random data sets for parallel analyses.
extraction	The factor extraction method. The options are: 'PAF' for principal axis / common factor analysis; 'PCA' for principal components analysis. 'image' for image analysis.
percentile	An integer indicating the percentile from the distribution of parallel analysis random eigenvalues. Suggested value: 95
corkind	The kind of correlation matrix to be used for the random data. The options are 'pearson', 'kendall', and 'spearman'.
verbose	Should detailed results be displayed in console? TRUE (default) or FALSE
factormodel	(Deprecated.) Use 'extraction' instead.

Details

This procedure for determining the number of components or factors involves comparing the eigenvalues derived from an actual data set to the eigenvalues derived from the random data. In Horn's original description of this procedure, the mean eigenvalues from the random data served as the comparison baseline, whereas the more common current practice is to use the eigenvalues that correspond to the desired percentile (typically the 95th) of the distribution of random data eigenvalues. Factors or components are retained as long as the i th eigenvalue from the actual data is greater than the i th eigenvalue from the random data. This function produces only random data eigenvalues and it does not take real data as input. See the RAWPAR function in this package for parallel analyses that also involve real data.

The PARALLEL function permits users to specify PCA or PAF or image as the factor extraction method. Principal components eigenvalues are often used to determine the number of common factors. This is the default in most statistical software packages, and it is the primary practice in the literature. It is also the method used by many factor analysis experts, including Cattell, who often examined principal components eigenvalues in his scree plots to determine the number of common factors. Principal components eigenvalues are based on all of the variance in correlation matrices, including both the variance that is shared among variables and the variances that are unique to the variables. In contrast, principal axis eigenvalues are based solely on the shared variance among the variables. The procedures are qualitatively different. Some therefore claim that the eigenvalues from one extraction method should not be used to determine the number of factors for another extraction method. The PAF option in the extraction argument for the PARALLEL function was included solely for research purposes. It is best to use PCA as the extraction method for regular data analyses.

Value

Random data eigenvalues

Author(s)

Brian P. O'Connor

References

Horn, J. L. (1965). A rationale and test for the number of factors in factor analysis. *Psychometrika*, 30, 179-185.

O'Connor, B. P. (2000). SPSS and SAS programs for determining the number of components using parallel analysis and Velicer's MAP test. *Behavior Research Methods, Instrumentation, and Computers*, 32, 396-402.

Zwick, W. R., & Velicer, W. F. (1986). Comparison of five rules for determining the number of components to retain. *Psychological Bulletin*, 99, 432-442.

Examples

```
PARALLEL(Nvars=15, Ncases=250, Ndatasets=100, extraction='PCA', percentile=95,
          corkind='pearson', verbose=TRUE)
```

PCA	<i>Principal components analysis</i>
-----	--------------------------------------

Description

Principal components analysis

Usage

```
PCA(data, corkind='pearson', Nfactors=NULL, Ncases=NULL, rotation='promax',
      ppower=3, verbose=TRUE, rotate)
```

Arguments

data	An all-numeric dataframe where the rows are cases & the columns are the variables, or a correlation matrix with ones on the diagonal. The function internally determines whether the data are a correlation matrix.
corkind	The kind of correlation matrix to be used if data is not a correlation matrix. The options are 'pearson', 'kendall', 'spearman', 'gamma', and 'polychoric'. Required only if the entered data is not a correlation matrix.
Nfactors	The number of components to extraction. If not specified, then the EMPKC procedure will be used to determine the number of components.
Ncases	The number of cases. Required only if data is a correlation matrix.
rotation	The factor rotation method for the analysis. The orthogonal rotation options are: 'varimax' (the default), 'quartimax', 'bentlerT', 'equamax', 'geominT', 'bifactorT', 'entropy', and 'none'. The oblique rotation options are: 'promax' (the default), 'quartimin', 'oblimin', 'oblimax', 'simplimax', 'bentlerQ', 'geominQ', 'bifactorQ', and 'none'.

ppower	The power value to be used in a promax rotation (required only if rotation = 'promax'). Suggested value: 3
verbose	Should detailed results be displayed in console? TRUE (default) or FALSE
rotate	(Deprecated.) Use 'rotation' instead.

Value

A list with the following elements:

loadingsNOROT	The unrotated factor loadings
loadingsROT	The rotated factor loadings
pattern	The pattern matrix
structure	The structure matrix
phi	The correlations between the factors
varexplNOROT1	The initial eigenvalues and total variance explained
varexplROT	The rotation sums of squared loadings and total variance explained for the rotated loadings
cormat_reprod	The reproduced correlation matrix, based on the rotated loadings
fit_coeffs	Model fit coefficients
communalities	The unrotated factor solution communalities
uniquenesses	The unrotated factor solution uniquenesses

Author(s)

Brian P. O'Connor

Examples

```
# the Harman (1967) correlation matrix
PCA(data_Harman, Nfactors=2, Ncases=305, rotation='oblimin', verbose=TRUE)

# Rosenberg Self-Esteem scale items
PCA(data_RSE, corkind='polychoric', Nfactors=2, rotation='bifactorQ', verbose=TRUE)

# NEO-PI-R scales
PCA(data_NEOPIR, corkind='pearson', Nfactors=5, rotation='promax', ppower = 4, verbose=TRUE)
```

POLYCHORIC_R	<i>Polychoric correlation matrix</i>
--------------	--------------------------------------

Description

Produces a polychoric correlation matrix

Usage

```
POLYCHORIC_R(data, method, verbose)
```

Arguments

data	An all-numeric data frame where the rows are cases & the columns are the variables. All values should be integers, as in the values for Likert rating scales.
method	(optional) The source package used to estimate the polychoric correlations: 'Revelle' for the psych package (the default); 'Fox' for the polycor package.
verbose	Should detailed results be displayed in console? TRUE (default) or FALSE

Details

Applying familiar factor analysis procedures to item-level data can produce misleading or uninterpretable results. Common factor analysis, maximum likelihood factor analysis, and principal components analysis produce meaningful results only if the data are continuous and multivariate normal. Item-level data almost never meet these requirements.

The correlation between any two items is affected by both their substantive (content-based) similarity and by the similarities of their statistical distributions. Items with similar distributions tend to correlate more strongly with one another than do with items with dissimilar distributions. Easy or commonly endorsed items tend to form factors that are distinct from difficult or less commonly endorsed items, even when all of the items measure the same unidimensional latent variable. Item-level factor analyses using traditional methods are almost guaranteed to produce at least some factors that are based solely on item distribution similarity. The items may appear multidimensional when in fact they are not. Conceptual interpretations of the nature of item-based factors will often be erroneous.

A common, expert recommendation is that factor analyses of item-level data (e.g., for binary response options or for ordered response option categories) or should be conducted on matrices of polychoric correlations. Factor analyses of polychoric correlation matrices are essentially factor analyses of the relations among latent response variables that are assumed to underlie the data and that are assumed to be continuous and normally distributed.

This is a cpu-intensive function. It is probably not necessary when there are > 8 item response categories.

By default, the function uses the polychoric function from William Revelle's' psych package to produce a full matrix of polychoric correlations. The function uses John Fox's hetcor function from the polycor package when requested or when the number of item response categories is > 8.

Value

The polychoric correlation matrix

Author(s)

Brian P. O'Connor

Examples

```
# Revelle polychoric correlation matrix for the Rosenberg Self-Esteem Scale (RSE)
POLYCHORIC_R(data_RSE, method = 'Revelle')

# Fox polychoric correlation matrix for the Rosenberg Self-Esteem Scale (RSE)
POLYCHORIC_R(data_RSE, method = 'Fox')
```

PROCRUSTES

Procrustes factor rotation

Description

Conducts Procrustes rotations of a factor loading matrix to a target factor matrix, and it computes the factor solution congruence and the root mean square residual (based on comparisons of the entered factor loading matrix with the Procrustes-rotated matrix).

Usage

```
PROCRUSTES(loadings, target, type, verbose)
```

Arguments

loadings	The loading matrix that will be aligned with the target.
target	The target loading matrix.
type	The options are 'orthogonal' or 'oblique' rotation.
verbose	Should detailed results be displayed in console? TRUE (default) or FALSE

Details

This function conducts Procrustes rotations of a factor loading matrix to a target factor matrix, and it computes the factor solution congruence and the root mean square residual (based on comparisons of the entered factor loading matrix with the Procrustes-rotated matrix). The orthogonal Procrustes rotation is based on Schonemann (1966; see also McCrae et al., 1996). The oblique Procrustes rotation is based on Hurley and Cattell (1962). The factor solution congruence is the Tucker-Wrigley-Neuhaus factor solution congruence coefficient (see Guadagnoli & Velicer, 1991; and ten Berge, 1986, for reviews).

Value

A list with the following elements:

loadingsPROC	The Procrustes-rotated loadings
congruence	The factor solution congruence after factor Procrustes rotation
rmsr	The root mean square residual
residmat	The residual matrix after factor Procrustes rotation

Author(s)

Brian P. O'Connor

References

Guadagnoli, E., & Velicer, W. (1991). A comparison of pattern matching indices. *Multivariate Behavior Research*, 26, 323-343.

Hurley, J. R., & Cattell, R. B. (1962). The Procrustes program: Producing direct rotation to test a hypothesized factor structure. *Behavioral Science*, 7, 258-262.

McCrae, R. R., Zonderman, A. B., Costa, P. T. Jr., Bond, M. H., & Paunonen, S. V. (1996). Evaluating replicability of factors in the revised NEO personality inventory: Confirmatory factor analysis versus Procrustes rotation. *Journal of Personality and Social Psychology*, 70, 552-566.

Schonemann, P. H. (1966). A generalized solution of the orthogonal Procrustes problem. *Psychometrika*, 31, 1-10.

ten Berge, J. M. F. (1986). Some relationships between descriptive comparisons of components from different studies. *Multivariate Behavioral Research*, 21, 29-40.

Examples

```
# RSE data
PCAoutput_1 <- PCA(data_RSE[1:150,], Nfactors = 2, rotation='promax', verbose=FALSE)

PCAoutput_2 <- PCA(data_RSE[151:300,], Nfactors = 2, rotation='promax', verbose=FALSE)

PROCRUSTES(target=PCAoutput_1$pattern, loadings=PCAoutput_2$pattern,
           type = 'orthogonal', verbose=TRUE)
```

Description

Parallel analysis of eigenvalues, with real data as input, for deciding on the number of components or factors.

Usage

```
RAWPAR(data, randtype, extraction, Ndatasets, percentile,
       corkind, corkindRAND, Ncases=NULL, eval_plot=TRUE, verbose, factormodel)
```

Arguments

data	An all-numeric data frame where the rows are cases & the columns are the variables, or a correlation matrix with ones on the diagonal. The function internally determines whether the data are a correlation matrix.
randtype	The kind of random data to be used in the parallel analysis: 'generated' for random normal data generation; 'permuted' for permutations of the raw data matrix.
extraction	The factor extraction method. The options are: 'PAF' for principal axis / common factor analysis; 'PCA' for principal components analysis. 'image' for image analysis.
Ndatasets	An integer indicating the # of random data sets for parallel analyses.
percentile	An integer indicating the percentile from the distribution of parallel analysis random eigenvalues to be used in determining the # of factors. Suggested value: 95
corkind	The kind of correlation matrix to be used if data is not a correlation matrix. The options are 'pearson', 'kendall', 'spearman', 'gamma', and 'polychoric'. Required only if the entered data is not a correlation matrix.
corkindRAND	The kind of correlation matrix to be used for the random data analyses. The options are 'pearson', 'kendall', 'spearman', 'gamma', and 'polychoric'. The default is 'pearson'.
Ncases	The number of cases upon which a correlation matrix is based. Required only if data is a correlation matrix.
eval_plot	Should a plot of the real and parallel analysis eigenvalues be produced? The options are TRUE (the default) or FALSE.
verbose	Should detailed results be displayed in console? TRUE (default) or FALSE
factormodel	(Deprecated.) Use 'extraction' instead.

Details

The parallel analysis procedure for deciding on the number of components or factors involves extracting eigenvalues from random data sets that parallel the actual data set with regard to the number of cases and variables. For example, if the original data set consists of 305 observations for each of 8 variables, then a series of random data matrices of this size (305 by 8) would be generated, and eigenvalues would be computed for the correlation matrices for the original, real data and for each of the random data sets. The eigenvalues derived from the actual data are then compared to the eigenvalues derived from the random data. In Horn's original description of this procedure, the mean eigenvalues from the random data served as the comparison baseline, whereas the more common current practice is to use the eigenvalues that correspond to the desired percentile (typically the 95th) of the distribution of random data eigenvalues. Factors or components are retained as long as the i th eigenvalue from the actual data is greater than the i th eigenvalue from the random data.

The RAWPAR function permits users to specify PCA or PAF or image as the factor extraction method. Principal components eigenvalues are often used to determine the number of common factors. This is the default in most statistical software packages, and it is the primary practice in the literature. It is also the method used by many factor analysis experts, including Cattell, who often examined principal components eigenvalues in his scree plots to determine the number of common factors. Principal components eigenvalues are based on all of the variance in correlation matrices, including both the variance that is shared among variables and the variances that are unique to the variables. In contrast, principal axis eigenvalues are based solely on the shared variance among the variables. The procedures are qualitatively different. Some therefore claim that the eigenvalues from one extraction method should not be used to determine the number of factors for another extraction method. The PAF option in the extraction argument for the PARALLEL function was included solely for research purposes. It is best to use PCA as the extraction method for regular data analyses.

Polychoric correlations are time-consuming to compute. While polychoric correlations should probably be specified for the real data eigenvalues when data consists of item-level responses, polychoric correlations probably should not be specified for the random data computations, even for item-level data. The procedure would take much time and it is unnecessary. Polychoric correlations are estimates of what the Pearson correlations would be had the real data been continuous. For item-level data, specify polychoric correlations for the real data eigenvalues (corkind='polychoric') and use the default for the random data eigenvalues (corkindRAND='pearson'). The option for using polychoric correlations for the random data computations (corkindRAND='polychoric') was provided solely for research purposes.

Value

A list with:

eigenvalues	the eigenvalues for the real and random data
NfactorsPA	the number of factors based on the parallel analysis

Author(s)

Brian P. O'Connor

References

Horn, J. L. (1965). A rationale and test for the number of factors in factor analysis. *Psychometrika*, 30, 179-185.

O'Connor, B. P. (2000). SPSS and SAS programs for determining the number of components using parallel analysis and Velicer's MAP test. *Behavior Research Methods, Instrumentation, and Computers*, 32, 396-402.

Zwick, W. R., & Velicer, W. F. (1986). Comparison of five rules for determining the number of components to retain. *Psychological Bulletin*, 99, 432-442.

Examples

```
# WISC data
```

```

RAWPAR(data_TabFid, randtype='generated', extraction='PCA', Ndatasets=100,
       percentile=95, corkind='pearson', verbose=TRUE)

# the Harman (1967) correlation matrix
RAWPAR(data_Harman, randtype='generated', extraction='PCA', Ndatasets=100,
       percentile=95, corkind='pearson', Ncases=305, verbose=TRUE)

# Rosenberg Self-Esteem scale items, using Pearson correlations
RAWPAR(data_RSE, randtype='permuted', extraction='PCA', Ndatasets=100,
       percentile=95, corkind='pearson', corkindRAND='pearson', verbose=TRUE)

# Rosenberg Self-Esteem scale items, using polychoric correlations
RAWPAR(data_RSE, randtype='generated', extraction='PCA', Ndatasets=100,
       percentile=95, corkind='polychoric', verbose=TRUE)

# NEO-PI-R scales
RAWPAR(data_NEOPIR, randtype='generated', extraction='PCA', Ndatasets=100,
       percentile=95, corkind='pearson', Ncases=305, verbose=TRUE)

```

RECODE	<i>Recode values in a vector</i>
--------	----------------------------------

Description

Options for changing the numeric values in a vector to new numeric values

Usage

```
RECODE(data, old = NULL, new = NULL, type = 'reverse', max_value = NULL,
       real_min = NULL, real_max = NULL, new_min = NULL, new_max = NULL)
```

Arguments

data	A numeric vector, typically consisting of item responses.
old	(optional) A vector of the values in data to be recoded, e.g., old = c(1,2,3,4).
new	(optional) A vector of the values that should replace the old values, e.g., new = c(0,0,1,1).
type	(optional) The type of recoding if "old" and "new" are not specified. The options are 'reverse' (for reverse coding) and 'new_range' (for changing the metric/range).
max_value	(optional) For type = 'reverse' coding only. It is the maximum possible value for data (an item). This option is included for when max(data) is not the maximum possible value for an item (e.g., when the highest response option was never used).
real_min	(optional) For type = 'new_range' coding only. The minimum possible value for data.

real_max	(optional) For type = 'new_range' coding only. The maximum possible value for data.
new_min	(optional) For type = 'new_range' coding only. The desired, new minimum possible value for data.
new_max	(optional) For type = 'new_range' coding only. The desired, new maximum possible value for data.

Details

When 'old' and 'new' are specified, the data values in the 'old' vector are replaced with the values in the same ordinal position in the 'new' vector, e.g., occurrences of the second value in 'old' in data are replaced with the second value in 'new' in data.

Regarding the type = 'new_range' option: Sometimes the items in a pool have different response option ranges, e.g., some on a 5-point scale and others on a 6-point scale. The type = 'new_range' option changes the metric/range of a specified item to a desired metric, e.g., so that scales scores based on all of the items in the pool can be computed. This alters item scores and the new item values may not be integers. Specifically, for each item response, the percent value on the real/used item is computed. Then the corresponding value on the desired new item metric for the same percentage is found.

Value

The recoded data values

Author(s)

Brian P. O'Connor

Examples

```
data <- c(1,2,3,4,1,2,3,4)
print(RECODE(data, old = c(1,2,3,4), new = c(1,1,2,2)) )

print(RECODE(data, type = 'reverse'))

# reversing coding the third item (Q3) of the Rosenberg Self-Esteem scale data
data_RSE_rev <- RECODE(data_RSE[, 'Q3'], type = 'reverse')
table(data_RSE_rev); table(data_RSE[, 'Q3'])

# changing the third item (Q3) responses for the Rosenberg Self-Esteem scale data
# from 0-to-4 to 1-to-5
data_RSE_rev <- RECODE(data_RSE[, 'Q3'], old = c(0,1,2,3,4), new = c(1,2,3,4,5))
table(data_RSE_rev); table(data_RSE[, 'Q3'])

# changing the metric/range of the third item (Q3) responses for the
# Rosenberg Self-Esteem scale data
data_RSE_rev <- RECODE(data_RSE[, 'Q3'], type = 'new_range',
                       real_min = 1, real_max = 4, new_min = 1, new_max = 5 )
table(data_RSE_rev); table(data_RSE[, 'Q3'])
```

ROOTFIT

*Factor fit coefficients***Description**

A variety of fit coefficients for the possible N-factor solutions in exploratory factor analysis

Usage

```
ROOTFIT(data, corkind='pearson', Ncases=NULL, extraction='PAF', verbose, factormodel)
```

Arguments

data	An all-numeric dataframe where the rows are cases & the columns are the variables, or a correlation matrix with ones on the diagonal. The function internally determines whether the data are a correlation matrix.
corkind	The kind of correlation matrix to be used if data is not a correlation matrix. The options are 'pearson', 'kendall', 'spearman', 'gamma', and 'polychoric'. Required only if the entered data is not a correlation matrix.
Ncases	The number of cases upon which a correlation matrix is based. Required only if data is a correlation matrix.
extraction	The factor extraction method. The options are: 'PAF' for principal axis / common factor analysis; 'PCA' for principal components analysis. 'ML' for maximum likelihood estimation.
verbose	Should detailed results be displayed in console? TRUE (default) or FALSE
factormodel	(Deprecated.) Use 'extraction' instead.

Details**Eigenvalue**

An eigenvalue is the variance of the factor. More specifically, an eigenvalue is the the variance of the linear combination of the variables for a factor. There are as many eigenvalues for a correlation or covariance matrix as there are variables in the matrix. The sum of the eigenvalues is equal to the number of variables. An eigenvalue of one means that a factor explains as much variance as one variable.

RMSR – Root Mean Square Residual (absolute fit)

RMSR (or perhaps more commonly, RMR) is an index of the overall badness-of-fit. It is the square root of the mean of the squared residuals (the residuals being the simple differences between original correlations and the correlations implied by the N-factor model). RMSR is 0 when there is perfect model fit. A value less than .08 is generally considered a good fit. A standardized version of the RMSR is often recommended over the RMSR in structural equation modeling analyses. This is because the values in covariance matrices are scale-dependent. However, the RMSR coefficient that is provided in this package is based on correlation coefficients (not covariances) and therefore does not have this problem.

GFI (absolute fit)

The GFI (McDonald, 1999) is an index of how closely a correlation matrix is reproduced by the factor solution. It is equal to $1.0 - \text{mean-squared residual} / \text{mean-squared correlation}$, ignoring the diagonals.

CAF (common part accounted for)

Lorenzo-Seva, Timmerman, & Kiers (2011): "We now propose an alternative goodness-of-fit index that can be used with any extraction method. This index expresses the extent to which the common variance in the data is captured in the common factor model. The index is denoted as CAF (common part accounted for)."

"A measure that expresses the amount of common variance in a matrix is found in the KMO (Kaiser, Meyer, Olkin) index (see Kaiser, 1970; Kaiser & Rice, 1974). The KMO index is commonly used to assess whether a particular correlation matrix R is suitable for common factor analysis (i.e., if there is enough common variance to justify a factor analysis)."

"Now, we propose to express the common part accounted for by a common factor model with q common factors as 1 minus the KMO index of the estimated residual matrix."

"The values of CAF are in the range [0, 1] and if they are close to zero it means that a substantial amount of common variance is still present in the residual matrix after the q factors have been extractioned (implying that more factors should be extractioned). Values of CAF close to one mean that the residual matrix is free of common variance after the q factors have been extractioned (i.e., no more factors should be extractioned)."

RMSEA - Root Mean Square Error of Approximation (absolute fit)

Schermelleh-Engel (2003): "The Root Mean Square Error of Approximation (RMSEA; Steiger, 1990) is a measure of approximate fit in the population and is therefore concerned with the discrepancy due to approximation. Steiger (1990) as well as Browne and Cudeck (1993) define a "close fit" as a RMSEA value $\leq .05$. According to Browne and Cudeck (1993), RMSEA values $\leq .05$ can be considered as a good fit, values between .05 and .08 as an adequate fit, and values between .08 and .10 as a mediocre fit, whereas values $> .10$ are not acceptable. Although there is general agreement that the value of RMSEA for a good model should be less than .05, Hu and Bentler (1999) suggested an RMSEA of less than .06 as a cutoff criterion."

Kenny (2020): "The measure is positively biased (i.e., tends to be too large) and the amount of the bias depends on smallness of sample size and df, primarily the latter. The RMSEA is currently the most popular measure of model fit. MacCallum, Browne and Sugawara (1996) have used 0.01, 0.05, and 0.08 to indicate excellent, good, and mediocre fit respectively. However, others have suggested 0.10 as the cutoff for poor fitting models. These are definitions for the population. That is, a given model may have a population value of 0.05 (which would not be known), but in the sample it might be greater than 0.10. There is greater sampling error for small df and low N models, especially for the former. Thus, models with small df and low N can have artificially large values of the RMSEA. For instance, a chi square of 2.098 (a value not statistically significant), with a df of 1 and N of 70 yields an RMSEA of 0.126. For this reason, Kenny, Kaniskan, and McCoach (2014) argue to not even compute the RMSEA for low df models."

Hooper (2008): "In recent years it has become regarded as "one of the most informative fit indices" (Diamantopoulos and Siguaw, 2000: 85) due to its sensitivity to the number of estimated parameters in the model. In other words, the RMSEA favours parsimony in that it will choose the model with the lesser number of parameters."

TLI – Tucker Lewis Index (incremental fit)

The Tucker-Lewis index, TLI, is also sometimes called the non-normed fit index, NNFI, or the Bentler-Bonett non-normed fit index, or RHO2. The TLI penalizes for model complexity.

Schermelleh-Engel (2003): "The (TLI or) NNFI ranges in general from zero to one, but as this index is not normed, values can sometimes leave this range, with higher (TLI or) NNFI values indicating better fit. A rule of thumb for this index is that .97 is indicative of good fit relative to the independence model, whereas values greater than .95 may be interpreted as an acceptable fit. An advantage of the (TLI or) NNFI is that it is one of the fit indices less affected by sample size (Bentler, 1990; Bollen, 1990; Hu & Bentler, 1995, 1998)."

Kenny (2020): "The TLI (and the CFI) depends on the average size of the correlations in the data. If the average correlation between variables is not high, then the TLI will not be very high."

CFI - Comparative Fit Index (incremental fit)

Schermelleh-Engel (2003): "The CFI ranges from zero to one with higher values indicating better fit. A rule of thumb for this index is that .97 is indicative of good fit relative to the independence model, while values greater than .95 may be interpreted as an acceptable fit. Again a value of .97 seems to be more reasonable as an indication of a good model fit than the often stated cutoff value of .95. Comparable to the NNFI, the CFI is one of the fit indices less affected by sample size."

Hooper (2008): "A cut-off criterion of $CFI \geq 0.90$ was initially advanced however, recent studies have shown that a value greater than 0.90 is needed in order to ensure that misspecified models are not accepted (Hu and Bentler, 1999). From this, a value of $CFI \geq 0.95$ is presently recognised as indicative of good fit (Hu and Bentler, 1999). Today this index is included in all SEM programs and is one of the most popularly reported fit indices due to being one of the measures least effected by sample size (Fan et al, 1999)."

Kenny (2020): "Because the TLI and CFI are highly correlated only one of the two should be reported. The CFI is reported more often than the TLI, but I think the CFI's penalty for complexity of just 1 is too low and so I prefer the TLI even though the CFI is reported much more frequently than the TLI."

MFI – (absolute fit)

An absolute fit index proposed by MacDonald and Marsh (1990) that does not depend on a comparison with another model.

AIC – Akaike Information Criterion (degree of parsimony index)

Kenny (2020): "The AIC is a comparative measure of fit and so it is meaningful only when two different models are estimated. Lower values indicate a better fit and so the model with the lowest AIC is the best fitting model. There are somewhat different formulas given for the AIC in the literature, but those differences are not really meaningful as it is the difference in AIC that really matters. The AIC makes the researcher pay a penalty of two for every parameter that is estimated. One advantage of the AIC, BIC, and SABIC measures is that they can be computed for models with zero degrees of freedom, i.e., saturated or just-identified models."

CAIC – Consistent Akaike Information Criterion (degree of parsimony index)

A version of AIC that adjusts for sample size. Lower values indicate a better fit.

BIC – Bayesian Information Criterion (degree of parsimony index)

Lower values indicate a better fit.

Kenny (2020): "Whereas the AIC has a penalty of 2 for every parameter estimated, the BIC increases the penalty as sample size increases. The BIC places a high value on parsimony (perhaps too high)."

SABIC – Sample-Size Adjusted BIC (degree of parsimony index)

Kenny (2020): "Like the BIC, the sample-size adjusted BIC or SABIC places a penalty for adding parameters based on sample size, but not as high a penalty as the BIC. Several recent simulation studies (Enders & Tofghi, 2008; Tofghi, & Enders, 2007) have suggested that the SABIC is a useful tool in comparing models."

Value

A list with eigenvalues & fit coefficients.

Author(s)

Brian P. O'Connor

References

Hooper, D., Coughlan, J., & Mullen, M. (2008). Structural Equation Modelling: Guidelines for Determining Model Fit. *Electronic Journal of Business Research Methods*, 6(1), 53-60.

Kenny, D. A. (2020). *Measuring model fit*. <http://davidaKenny.net/cm/fit.htm>

McDonald, R. P. (1999). *Test theory: A unified treatment*. Mahwah, NJ: Lawrence Erlbaum Associates, Publishers.

Lorenzo-Seva, U., Timmerman, M. E., & Kiers, H. A. (2011). The Hull method for selecting the number of common factors. *Multivariate Behavioral Research*, 46, 340-364.

Schermelleh-Engel, K., & Moosbrugger, H. (2003). Evaluating the fit of structural equation models: Tests of significance and descriptive goodness-of-fit measures. *Methods of Psychological Research Online*, Vol.8(2), pp. 23-74.

Tabachnick, B. G., & Fidell, L. S. (2019). *Using multivariate statistics* (pp. 560-564). New York, NY: Pearson.

Examples

```
# the Harman (1967) correlation matrix
ROOTFIT(data_Harman, Ncases = 305, extraction='ml')
ROOTFIT(data_Harman, Ncases = 305, extraction='paf')
ROOTFIT(data_Harman, Ncases = 305, extraction='pca')

# RSE data
ROOTFIT(data_RSE, corkind='pearson', extraction='ml')
ROOTFIT(data_RSE, corkind='pearson', extraction='paf')
ROOTFIT(data_RSE, corkind='pearson', extraction='pca')

# NEO-PI-R scales
ROOTFIT(data_NEOPIR, corkind='pearson', extraction='ml')
ROOTFIT(data_NEOPIR, corkind='pearson', extraction='paf')
ROOTFIT(data_NEOPIR, corkind='pearson', extraction='pca')
```

SALIENT*Salient loadings criterion for the number of factors*

Description

Salient loadings criterion for determining the number of factors, as recommended by Gorsuch. Factors are retained when they consist of a specified minimum number (or more) variables that have a specified minimum (or higher) loading value.

Usage

```
SALIENT(data, salvalue=.4, numsals=3, max_cross=NULL, min_eigval=.7, corkind='pearson',
       extraction = 'paf', rotation='promax', loading_mat = 'structure',
       ppower = 3, iterpaf=100, Ncases=NULL, verbose=TRUE, factormodel, rotate)
```

Arguments

data	An all-numeric dataframe where the rows are cases & the columns are the variables, or a correlation matrix with ones on the diagonal. The function internally determines whether the data are a correlation matrix.
salvalue	(optional) The loading value that is considered salient. Default = .40. This can also be a vector of up to three values, e.g., salvalue = c(.4, .5, .6).
numsals	(optional) The required number of salient loadings for a factor. Default = 3. This can also be a vector of up to three values, e.g., numsals = c(3, 2, 1).
max_cross	(optional) The maximum value for cross-loadings.
min_eigval	(optional) The minimum eigenvalue for including a factor in the analyses. Default = .7
corkind	(optional) The kind of correlation matrix to be used if data is not a correlation matrix. The options are 'pearson', 'kendall', 'spearman', 'gamma', and 'polychoric'. Required only if the entered data is not a correlation matrix.
extraction	(optional) The factor extraction method for the analysis. The options are 'pca', 'paf' (the default), 'ml', 'image', 'minres', 'uls', 'ols', 'wls', 'gls', 'alpha', and 'fullinfo'.
rotation	(optional) The factor rotation method for the analysis. The orthogonal rotation options are: 'varimax' (the default), 'quartimax', 'bentlerT', 'equamax', 'geominT', 'bifactorT', 'entropy', and 'none'. The oblique rotation options are: 'promax' (the default), 'quartimin', 'oblimin', 'oblimax', 'simplimax', 'bentlerQ', 'geominQ', 'bifactorQ', and 'none'.
iterpaf	(optional) The maximum number of iterations for paf. Default value = 100
loading_mat	(optional) The kind of factor rotation matrix for an oblique rotation. The options are 'structure' (the default) or 'pattern'.

ppower	(optional) The power value to be used in a promax rotation (required only if rotation = 'promax'). Default value = 3
Ncases	The number of cases. Required only if data is a correlation matrix.
verbose	(optional) Should detailed results be displayed in console? TRUE (default) or FALSE
factormodel	(Deprecated.) Use 'extraction' instead.
rotate	(Deprecated.) Use 'rotation' instead.

Details

In this procedure for determining the number of factors, factors are retained when each factor has at least a specified minimum number variables (e.g., 3) that have loadings that are greater than or equal to a specified minimum loading value (e.g., .40). Factor are considered trivial when they do not contain a sufficient number of salient loadings (Gorsuch, 1997, 2015; Boyd, 2011).

The procedure begins by extracting and rotating (if requested) an excessive number of factors. If the initial factor loadings do not meet the specified criteria, then the factor analysis is conducted again with one less factor and the loadings are again examined to determine whether the factor loadings meet the specified criteria. The procedure stops when a loading matrix meets the criteria, in which case the number of columns in the loading matrix is the number of factors according to the salient loadings criteria.

The initial, excessive number of factors for the procedure is determined using the **min_eigval** argument (for minimum eigenvalue). The default is .70, which can be adjusted (raised) when analyses produce an error caused by there being too few variables.

Although there is no consensus on what constitutes a 'salient' loading, an absolute value of .40 is common in the literature.

There are different versions of the salient loadings criterion method, which has not been extensively tested to date. The procedure is nevertheless considered promising by Gorsuch and others.

Some versions involve the use of multiple salient loading values, each with its own minimum number of variables. This can be done in the SALIENT function by providing a vector of values for the **salvalue** argument and a corresponding vector of values for the **numsals** argument. The maximum number of possible values is three, and there should be a logical order in the values, i.e., increasing values for salvalue and decreasing values for numsals.

It is also possible to place a restriction of the maximum value of the cross-loadings for the salient variables, e.g., requiring that a salient loading is not accompanied by cross-loadings on other variables that are greater than .15. Use the **max_cross** argument for this purpose, although it may be difficult to claim that cross-loadings should be small when the factors are correlated.

Value

The number of factors according to the salient loadings criterion.

Author(s)

Brian P. O'Connor

References

Boyd, K. C. (2011). Factor analysis. In M. Stausberg & S. Engler (Eds.), *The Routledge Handbook of Research Methods in the Study of Religion* (pp. 204-216). New York: Routledge.

Gorsuch, R. L. (1997). Exploratory factor analysis: Its role in item analysis. *Journal of Personality Assessment*, 68, 532-560.

Gorsuch, R. L. (2015). *Factor analysis*. Routledge/Taylor & Francis Group.

Examples

```
# the Harman (1967) correlation matrix
SALIENT(data_Harman, salvalue=.4, numsals=3, Ncases=305)

# Rosenberg Self-Esteem scale items, using Pearson correlations
SALIENT(data_RSE, salvalue=.4, numsals=3, corkind='pearson')

# NEO-PI-R scales
SALIENT(data_NEOPIR, salvalue = c(.4, .5, .6), numsals = c(3, 2, 1), extraction = 'paf',
        rotation='promax', loading_mat = 'pattern')
```

SCREE_PLOT

Scree plot of eigenvalues

Description

Produces a scree plot of eigenvalues for raw data or for a correlation matrix.

Usage

```
SCREE_PLOT(data, corkind, Ncases, verbose)
```

Arguments

data	An all-numeric dataframe where the rows are cases & the columns are the variables, or a correlation matrix with ones on the diagonal. The function internally determines whether the data are a correlation matrix.
corkind	The kind of correlation matrix to be used if data is not a correlation matrix. The options are 'pearson', 'kendall', 'spearman', 'gamma', and 'polychoric'. Required only if the entered data is not a correlation matrix.
Ncases	The number of cases for a correlation matrix. Required only if the entered data is a correlation matrix.
verbose	Should detailed results be displayed in console? TRUE (default) or FALSE

Value

totvarexpl	The eigenvalues and total variance explained
------------	--

Author(s)

Brian P. O'Connor

Examples

```
# Field's RAQ factor analysis data
SCREE_PLOT(data_Field, cokind='pearson')

# the Harman (1967) correlation matrix
SCREE_PLOT(data_Harman)

# Rosenberg Self-Esteem scale items
SCREE_PLOT(data_RSE, cokind='polychoric')

# NEO-PI-R scales
SCREE_PLOT(data_RSE)
```

SESCREE

Standard Error Scree test

Description

This is a linear regression operationalization of the scree test for determining the number of components. The results are purportedly identical to those from the visual scree test. The test is based on the standard error of estimate values that are computed for the set of eigenvalues in a scree plot. The number of components to retain is the point where the standard error exceeds $1/m$, where m is the numbers of variables.

Usage

```
SESCREE(data, Ncases=NULL, cokind, verbose=TRUE)
```

Arguments

data	An all-numeric dataframe where the rows are cases & the columns are the variables, or a correlation matrix with ones on the diagonal. The function internally determines whether the data are a correlation matrix.
Ncases	The number of cases. Required only if data is a correlation matrix.
cokind	The kind of correlation matrix to be used if data is not a correlation matrix. The options are 'pearson', 'kendall', 'spearman', 'gamma', and 'polychoric'. Required only if the entered data is not a correlation matrix.
verbose	Should detailed results be displayed in console? TRUE (default) or FALSE

Value

The number of components according to the Standard Error Scree test.

Author(s)

Brian P. O'Connor

References

Zoski, K., & Jurs, S. (1996). An objective counterpart to the visual scree test for factor analysis: the standard error scree test. *Educational and Psychological Measurement*, 56(3), 443-451.

Examples

```
# the Harman correlation matrix
SESCREE(data_Harman, Ncases=305, verbose=TRUE)

# the Rosenberg Self-Esteem Scale (RSE) using Pearson correlations
SESCREE(data_RSE, corkind='pearson', verbose=TRUE)

# the Rosenberg Self-Esteem Scale (RSE) using polychoric correlations
SESCREE(data_RSE, corkind='polychoric', verbose=TRUE)

# the NEO-PI-R scales
SESCREE(data_NEOPIR, verbose=TRUE)
```

SMT

Sequential chi-square model tests

Description

A test for the number of common factors using the likelihood ratio test statistic values from maximum likelihood factor analysis estimations.

Usage

```
SMT(data, corkind, Ncases=NULL, verbose)
```

Arguments

data	An all-numeric data frame where the rows are cases & the columns are the variables, or a correlation matrix with ones on the diagonal. The function internally determines whether the data are a correlation matrix.
corkind	The kind of correlation matrix to be used if data is not a correlation matrix. The options are 'pearson', 'kendall', 'spearman', 'gamma', and 'polychoric'. Required only if the entered data is not a correlation matrix.
Ncases	The number of cases. Required only if data is a correlation matrix.
verbose	Should detailed results be displayed in console? TRUE (default) or FALSE

Details

From Auerswald & Moshagen (2019):

"The fit of common factor models is often assessed with the likelihood ratio test statistic (Lawley, 1940) using maximum likelihood estimation (ML), which tests whether the model-implied covariance matrix is equal to the population covariance matrix. The associated test statistic asymptotically follows a Chi-Square distribution if the observed variables follow a multivariate normal distribution and other assumptions are met (e.g., Bollen, 1989). This test can be sequentially applied to factor models with increasing numbers of factors, starting with a zero-factor model. If the Chi-Square test statistic is statistically significant (with e.g., $p < .05$), a model with one additional factor, in this case a unidimensional factor model, is estimated and tested. The procedure continues until a nonsignificant result is obtained, at which point the number of common factors is identified.

"Simulation studies investigating the performance of sequential Chi-Square model tests (SMT) as an extraction criterion have shown conflicting results. Whereas some studies have shown that SMT has a tendency to overextraction (e.g., Linn, 1968; Ruscio & Roche, 2012; Schonemann & Wang, 1972), others have indicated that the SMT has a tendency to underextraction (e.g., Green et al., 2015; Hakstian et al., 1982; Humphreys & Montanelli, 1975; Zwick & Velicer, 1986). Hayashi, Bentler, and Yuan (2007) demonstrated that overextraction tendencies are due to violations of regularity assumptions if the number of factors for the test exceeds the true number of factors. For example, if a test of three factors is applied to samples from a population with two underlying factors, the likelihood ratio test statistic will no longer follow a Chi-Square distribution. Note that the tests are applied sequentially, so a three-factor test is only employed if the two-factor test was incorrectly significant. Therefore, this violation of regularity assumptions does not decrease the accuracy of SMT, but leads to (further) overextraction if a previous test was erroneously significant. Additionally, this overextraction tendency might be counteracted by the lack of power in simulation studies with smaller sample sizes. The performance of SMT has not yet been assessed for non-normally distributed data or in comparison to most of the other modern techniques presented thus far in a larger simulation design." (p. 475)

Value

A list with the following elements:

NfactorsSMT	number of factors according to the SMT
pvalues	eigenvalues, chi-square values, & pvalues

Author(s)

Brian P. O'Connor

References

Auerswald, M., & Moshagen, M. (2019). How to determine the number of factors to retain in exploratory factor analysis: A comparison of extraction methods under realistic conditions. *Psychological Methods*, 24(4), 468-491.

Examples

```
# the Harman (1967) correlation matrix
```

```
SMT(data_Harman, Ncases=305, verbose=TRUE)

# Rosenberg Self-Esteem scale items, using Pearson correlations
SMT(data_RSE, corkind='polychoric', verbose=TRUE)

# NEO-PI-R scales
SMT(data_NEOPIR, verbose=TRUE)
```

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